Program Cover Document- MAT 255: Perspectives on the Development of Math

I. Basic Course Information

MAT 255: Perspectives on the Development of Math is designed to satisfy the Liberal Learning outcomes in the Domain of Human Inquiry/ Worldviews and Ways of Knowing.

This course will explore topics from the history of pre-modern mathematics like the development of number systems, the relationship of Greek and Mesopotamian algebra, the development of the proof process and paradoxes of the infinite.

The course will explore mathematics as it was created and it will convey the intellectual excitement experienced by the human mathematics-makers in their historical problem-situations. In this course we intentionally deemphasize the timeless abstract purity of mathematics to confront students with the fact that mathematics is a historically and socially situated human endeavor that not only served as the language of science for hundreds of years but also had a crucial role in shaping philosophical views of man and the world.

II. Learning Goals

The primary goal MAT 255: Perspectives on the Development of Math is to investigate the historical development of mathematics as a human activity, influenced by individual personalities, human interaction, social contexts and cultural contexts.

Students will gain an overall familiarity with the history of mathematics and will be able to place events at the proper point of the development of the subject. They will be able to read the secondary literature about topics discussed in class and they will be able to critically interact with it.

The course is a writing intensive course designed to improve students' written communication skills in a technical setting.

It is a required course for mathematics education majors following the recommendation of the Conference Board of Mathematical Sciences and the NCATE accreditation guidelines. Our goal is to show some examples of how the history of mathematics could be used in teaching and to inspire prospective teachers to introduce a multicultural and interdisciplinary perspective into their teaching.

It is also our hope that students will recognize that studying the history of mathematics is an alternative and potentially deliberating way to continue to learn mathematics throughout their lives.

III. Student Assessment

Writing assignments may include essays, research papers, biographies, article reviews, or other projects.

Content knowledge will be tested through a combination of quizzes and examinations.

Regular individual or group homework assignments will aid students to understand the mathematical concepts discussed in class and will help them to become familiar with problem-solving techniques of the past. Students will receive

regular feedback on homework.

Students may be asked to complete reading assignments and participate subsequently in class discussions at the discretion of the instructor.

IV. Learning activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance at lectures, in-class activities, reading assignments (discussion of secondary sources, analysis of original sources), individual or group presentations, graded homework assignments, keeping a reading response journal, short writing assignments (review of an article, critique of a web-site) , or substantial research papers on a selected topic.

Departmental Course Syllabus--MAT 255 Perspectives on the Development of Math

Course information

A. Purpose statement

The effect of the education machine is frequently to leave the impression that mathematicians are a peculiar hybrid of pedant and prophet who are either laboriously proving obvious things or they are conjuring up obscure formulae from nowhere that are to be memorized and cast like spells over problems by their students. The excepted style of exposition in lectures and texts is a somewhat monotonous succession of definitions and theorems that leaves little room for creativity. Topics are treated from the modern point of view and students are never confronted with the fact that mathematics is a historically and socially situated human endeavor. The challenge of this course is to allow the student to share something of the conflicts, creative tensions, and intellectual excitement experienced by the human mathematics-makers in their historical problem-situations when these venerable concepts were just beginning to make their names, to communicate the exhilaration of the hunt, not just the "state-of-the art": the adventures of the journey, not just the landmarks, monuments, and trophies. This class offers an antidote to the shiny mathematical exposition presented in many books and reveals some of the confusions, false starts, and misunderstandings inherent in the work of even very well respected mathematicians. It is an alternative, indirect, but potentially liberating way to learn mathematics.

The evolution of mathematical ideas is a complex one and it is certainly not the story of the unfolding of modern understandings of mathematics. Mathematical ideas often come with overtones from other disciplines, e.g. Greek philosophy influenced notions of number and proof. Religious beliefs influenced the development of mathematical concepts (the notion of "probability" could not surface until everything was believed to be minutely controlled by the will of God) and also the acceptance of mathematical theories e.g. the discovery of incommensurables, supposedly caused a spiritual crisis for the Pythagoreans. It is the aim of this course to point out connections between different areas of human inquiry and to explore as widely as possible the entire cultural climate that leads to the crystallization of certain mathematical ideas and often to multiple discoveries, e.g. non-Euclidean geometry. It is also the goal of this course to point out cross-cultural contributions to a given problem e.g. solving equations, from the Babylonians through Euclid to Al-Khwarismi, and to point out how socio-economical factors contributed to creating prosperous research centers.

Mathematics has a comfortable characteristic: the words "right" and "wrong" apply to it more easily than to any other subject. The study of its history however, is more complicated than that. In studying the history of mathematics we have to identify the rightness of "wrong" ideas (infinitesimals, complex numbers), as well as the wrongness of "correct" ideas (Aristotle's theory of motion). The question often times is not what is right and what is wrong but the incommensurability of different paradigms. Mathematics does not grow with a monotonous increase in the number of indubitably established theorems but through guesses and criticisms following the "logic of proofs and

refutations" (Lakatos). In this class students will investigate how mathematical theories are being constantly challenged and defended e.g. the development of set theory, leading to higher and higher level of correctness but also to abstract ideas that seem entirely detached from common sense.

In this course we also wish to emphasize the central role mathematics has played in shaping of views of man and the world held not just by scientists but by everyone educated in the western tradition. The influence of Greek geometry on Plato's theory of knowledge and cosmology, the advocating of the mathematical method from Aristotle to Spinoza and the discovery of non-Euclidean geometry leading all sciences to replace their claim of speaking directly about reality with the more modest claim of creating only different models of it are but a few examples for the vastness of that influence.

This course is a required course for mathematics education majors. The relevance of the history of mathematics in enlivening and illuminating the teaching of mathematics for all the above mentioned reasons is widely accepted. The history of mathematics is also rich in human anecdote providing a colorful backdrop to the study of mathematics. Archimedes' murder; Galois' last night; Descartes' fly; Newton's apple; all are powerful symbols of the vitality of mathematics and its compelling grip on the human mind. There are many other uses of history in teaching beyond the occasional sprinkling of anecdotes, e.g. the use of Babylonian geometric algebra in the teaching of quadratic equations and it is the goal of this course to educate the prospective teacher about them. The course should also inspire our education students to introduce a multicultural and interdisciplinary perspective into their teaching.

B) Course description

This course is not an exhaustive survey of the entire history of mathematics. The course develops a skeletal outline of the history of mathematics but provides the instructor with complete freedom to concentrate on the history of a specific area of mathematics or to focus on certain selected periods in history. Some topics that are suitable for discussion in this course are listed below to provide examples:

Ancient mathematics: how and why ancient people solved various types of mathematical problems, what the social context was and who were the people who "did" mathematics, especially in Egypt and Mesopotamia.

Greek mathematics: the relationship of Greek and Mesopotamian algebra, the development of the proof process, the discovery of incommensurability.

Trigonometry and its travels: the growth and travels of trigonometric ideas to measure the heavens from Greece and Egypt to India, then China, to the Middle East, and finally back to Europe .

Cubic equations: Cubic equations were solved first using geometric techniques by Omar Khayyam in the eleventh century and then algebraically by various Italian mathematicians in the sixteenth century. One can investigate their methods and then see how the concept of complex numbers grew out of the algebraic solution techniques.

Infinity: Zeno's paradoxes, paradoxes of the infinite, Cantor's definition of cardinality, Russell's definition of number, the role of paradoxes in mathematics.

The concept of number: development of number systems and number words, zero, the history of negative and irrational numbers.

C) Prerequisites: None

II. Learning goals

A. Content goals

- Students will recognize mathematics as a humanistic subject as described in the purpose statement.
- Students will gain an overall familiarity with the history of mathematics and will be able to place events at the proper point of the development of the subject.
- Student will understand the mathematical details of the historical expositions.

B. Performance goals

- Students will be able to read the secondary literature in the history of mathematics and interact critically with it.
- Students will be able to research a selected topic on their own and express the results of their research in written form at the level of a good science writer.
- Students, future educators especially, will recognize that the history of mathematics provides an enjoyable avenue to continue to learn mathematics throughout their lives.
- Prospective teachers will recognize the importance of using history in teaching.

III. Student assessment

A syllabus should clearly describe the schedule for these assessment tools, the criteria that will be used to evaluate student performance, and how the grades will be calculated. Assessment of the success of students will be done through a combination of the following methods at the instructor's discretion:

- Content knowledge will be tested through quizzes and examinations. Homework assignments will be assigned regularly to further develop students' understanding of the mathematical concepts, problems and methods discussed in class. Students are provided with feedback regularly on their homework to correct their misunderstandings.
- Students will complete reading assignments before each class. They will answer questions about the readings or hand in a brief review of the assignment.
- Students may be required to write a term paper on a selected topic.

IV. Learning activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance at lectures, in-class activities, reading assignments (discussion of secondary sources, analysis of original sources), individual or group presentations, graded homework assignments, keeping a reading response journal, short writing assignments (review of an article, critique of a web-site). Students may be required to write a substantial research paper on a selected topic.

A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students.