Program Cover Document --- MAT 305: Abstract Algebra

I. Basic Course Information

MAT 305: Abstract Algebra is primarily a junior/senior level course. It is scheduled for two 80-minute meetings each week. Its prerequisites are MAT 200 and MAT 205.

Abstract Algebra is one of the fundamental disciplines in modern mathematics. Starting in antiquity with everyday practical problems, it developed in the eighteenth and nineteenth centuries into a theoretical discipline unifying fields like number theory, the theory of equations and geometry. Historically, much of the theory in abstract algebra was developed independently by number theorists and by geometers. The outcome of these efforts was a discipline that interprets the foundations of many areas in mathematics.

II. Learning Goals

The primary goal of MAT 305: Abstract Algebra is to provide students with an introduction to one of the fundamental subjects in modern mathematics. Students will be introduced to the basic algebraic structures of groups, rings, and fields and the abstract notions that unify these diverse mathematical objects.

The Abstract Algebra course is designed to meet the needs of all the students in the class. The course will give all the necessary background in abstract algebra for students like liberal arts math majors (MATA) who wish to study more advanced topics in abstract algebra or students from the sciences who will use the algebraic structures in their research. At the same time, Abstract Algebra is a required course for future secondary school mathematics teachers. It has been developed following the guidelines of the Conference Board of Mathematical Sciences to insure that a student in the teacher preparation program will have a strong foundation in algebra.

Topics are chosen to cover historically important problems in algebra, to give an understanding of the subject's historical development and to illustrate the common underlying algebraic structure beneath seemingly unrelated subjects. The basic algebraic structures are groups, rings, integral fields, and fields. Fundamental applications of their basic properties will be seen by students in the arithmetic of the integers, the integers mod n, the rational numbers, the real numbers, the complex numbers and the transformations of geometric objects such as regular polygons.

An additional learning goal of Abstract Algebra is the further development of both a student's abstract reasoning ability and a student's ability to read, write, and understand proofs. The level of proofs in Abstract Algebra is at a high level and builds upon the techniques of proof that a student has learned in MAT 200 and MAT 205.

III. Student Assessment

This course is intended to be highly homework intensive. Weekly reading and writing assignments will constantly provide the instructor with information on the progress of individual students. At the same time students will receive weekly feedback on their work and their progress. A combination of quizzes and tests throughout the course will provide further valuable information both for the instructor and the individual students.

Assessment of the success of Abstract Algebra in meeting its learning goals will be done through a combination of student performance in the course and in their subsequent use of algebra in higher-level courses such as the senior capstone seminar.

IV. Learning Activities

The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, quizzes, tests and final exam.

Departmental Course Syllabus --- MAT 305: Abstract Algebra

Introduction: A typical syllabus for Abstract Algebra follows this sheet. Any syllabus for Abstract Algebra should include the points listed below (the required course requirement sections).

I. Basic information on course and instructor

A. Purpose statement:

Abstract Algebra is one of the fundamental disciplines in modern mathematics. Starting in antiquity with everyday practical problems, it developed in the eighteenth and nineteenth centuries into a theoretical discipline unifying fields like number theory, the theory of equations and geometry. Historically, much of the theory in abstract algebra was developed independently by number theorists and by geometers. The outcome of these efforts was a discipline that interprets the foundations of many areas in mathematics.

The arithmetic of the integers, the integers mod n, the rational numbers, the real numbers, the complex numbers as well as the transformations of certain geometric objects are just examples of algebraic structures. The basic underlying algebraic structures groups, rings, integral domains and fields are the abstract notions that unify these diverse mathematical objects.

Math 305: Abstract Algebra will first provide the students with an introduction to all the basic algebraic structures, groups, rings, integral domains and fields with numerous concrete applications and their basic properties. In the case of groups the symmetries of regular polygons and the groups of symmetries in general will be studied . In the case of rings, the ring of integers and the rings of polynomials over a field, will give the students an understanding of the historical development of the subject. The algebraic similarities between the ring of integers and the ring of polynomials will be emphasized. The construction of the field of rational numbers will be studied. In addition the field of real numbers, the field of complex numbers and the field of integers mod a prime p will be used when studying solutions of polynomial equations.

B. Course description: An introduction to groups, rings, integral domains, and fields. Additional topics include groups of symmetries, rings of integers, roots of polynomial equations, and construction of the rational numbers.

C. Course prerequisites: MAT 200 and MAT 205.

II. Learning goals

A. Content goals: Students will gain proficiency with many basic topics in abstract algebra. The course will introduce students to groups, rings, integral domains, and fields. They will learn about symmetry groups and the roots and factorization of polynomials defined over various rings and fields. They will learn about the construction of various number systems.

B. Performance goals: At the completion of the course, students should demonstrate competence with abstract algebra concepts. A successful abstract algebra student should be able to work with symmetry groups and understand their usefulness. They should be able to do algebra calculations involving polynomials over various number systems. They should be comfortable working with unusual groups and rings. They should understand the connection between the abstract systems they have studied and the concrete properties of the number systems they are familiar with. They should understand the connection between algebra and the solution of problems from other fields of mathematics.

III. Student assessment

A. Assessment Plan: This course is intended to be highly homework intensive. Weekly reading and writing assignments will constantly provide the instructor with information on the progress of individual students. At the same time students will receive weekly feedback on their work and their progress. A combination of quizzes and tests throughout the course will provide further valuable information both for the instructor and the individual students. A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.
B. Rationale: Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.
C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to use their knowledge in the future and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

A. Summary of learning activities: The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, quizzes, tests and final exam.
B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester. As a general rule, it is expected that each of the major topics of groups, rings, and fields will be given equal emphasis.
C. Rationale By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of abstract algebra and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students continual regular feedback on their work.

Math 305

Course Syllabus
Fall'03

Instructor: A. Papantonopoulou
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I. Learning Goals

A. Content

1. The following fundamental algebraic structures and their corresponding substructures will be defined and studied: groups, rings, integral domains and fields. Their basic properties will be derived. Numerous examples of these algebraic structures from number theory, the theory of equations and geometry will be studied.
2. In the case of groups special emphasis will be given on the groups of permutations and the dihedral groups of symmetries of regular polygons. In the case of rings the ring of polynomials over a field will be studied in depth.

B. Performance

1. Students will be expected to master the basic notions described in the content. They will be expected to recognize the underlying algebraic structures in a given setting or problem. They should be able to derive elementary properties of all these algebraic structures.
2. Students will be expected to be able to explain in writing as well as orally to their peers the studied notions and their properties

II. Student Assessment

A. There will be weekly reading assignments and weekly written homework assignments due every week. There will be two major tests during the semester and one final comprehensive exam at the end of the semester.

B. The weekly assignments will structure the course. They will enable the students to work continuously on the subject throughout the semester. Each major test will come at the conclusion of one big topic and will allow the students to review the material covered up to that point before proceeding with the new big topic.

C. The weekly written assignments and the two tests will be read , corrected, graded and returned to the students with comments during the following class meeting. The comments will indicate where the student has made incorrect assumptions or false reasoning. The student will be expected to redo those problems till the correct steps are found.

D. Grading Policy. The final grade will be determined as follows: Weekly assignments (20%), Test 1 (20%), Test 2, (20%), Final Exam (40%).

III. Learning Activities

1. Students are expected to participate in study groups. The study groups will be organized the first day of class. Students will be expected to meet with their study group at least for two hours every week. They should have done their weekly reading and try the homework assignment of that week before meeting their peers in the study

group. Students should be responsible for the preparation on that week's assignment of every member of their study group.

2. Students should be prepared to explain their work to the rest of the class every week.
3. Students are expected to bring their study groups questions on the reading or the written assignment at the beginning of each class period.

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