#### Program Cover Document—MAT 317: Linear Programming

#### I. Basic Course Information

Linear Programming deals with the problem of maximizing or minimizing a linear function subject to a set of linear constraints. Areas as diverse as finance, the military and medicine all extensively exploit the mathematical techniques and theory of linear programming. The field is important to many businesses because it allows complex problems to be represented in the form of mathematical models. These models can be solved in reasonable amounts of time through the use of efficient algorithms. Major corporations have Operations Research departments devoted to the applications of Linear Programming. The course will be scheduled for two 80-minute lecture periods, and a third period may be used at the Instructor's discretion for a computer lab, help session, etc. The course is an upper level option for both Majors and Minors in Mathematics, Mathematics Education or Statistics. The prerequisite for the course is MAT 205.

This course provides an introduction to the algorithms, theoretical underpinnings, and applications of linear programming. Topics may include: the simplex method for solving a linear program (LP), the geometry of LPs, variants of the simplex method, constructing mathematical models using LPs, duality theory, sensitivity analysis, integer programming, transportation and trans-shipment models, network models, program management models, and solving LPs using modern software packages (Excel, LINDO, LINGO, AMPL or MATLAB). Connections to Linear Algebra will be highlighted where relevant.

#### **II. Learning Goals**

The field of Operations Research consists of a broad collection of techniques with the main goal of solving optimization problems. This course is devoted to the study of Linear Programming (LP), the subset of Operations Research that uses *deterministic* mathematical techniques to solve optimization problems. Applications of linear programming include transportation problems, flight scheduling, medication dosage, corporate planning, inventory control, production scheduling and many others. The primary goal of this course is to have the student understand the mathematical underpinnings of LP techniques and to apply the techniques to real problems. In order to understand the mathematics of LP, both the theory behind the techniques and the paper-and-pencil execution of the techniques will be considered. However, in order to solve most real-world examples, LP software is required. Therefore, in this course we will also learn how to utilize a number of different LP software packages. Finally, students will investigate applications of LP using the Interfaces publication which uses mathematical models to solve real-world, large-scale optimization problems. Upon completion of the course, the student should have a foundation in the following areas:

- 1. Theory of Linear Programming
  - a) Learning the general characteristics of optimization problems with a linear objective function and linear constraints
  - b) Understanding why and under what conditions the simplex method and its variants can find the solution to a linear programming problem

- c) Extending the theory to linear programming problems that require integer solutions and dealing with the resulting increased complexity
- 2. Applications of Linear Programming
  - a) Identifying problems which can be solved through optimization
  - b) Selecting a modeling technique
  - c) Understanding the data collection that would be required
  - d) Constructing and validating LP models
  - e) Evaluating model results
- 3. Using Linear Programming software packages (such as Excel, LINDO, LINGO, AMPL or MATLAB) to efficiently solve LPs

#### **III. Student Assessment**

Students will receive regular feedback on their work through a combination of homework, quizzes, projects and examinations. Regular textbook-based homework will be used to assess student understanding of the theory and methods presented in class. Homework will also be used to assess the student's facility with the various software packages introduced.

Project(s) will be assigned during the semester to assess how effectively students can assimilate the various techniques taught in the class to solve larger, more complex problems. Formal documentation of the project and its results are important in assessing the student's thinking and writing skills as well as his/her ability to discover a creative and mathematically sound solution.

A midterm and a final exam will be used to assess overall understanding of the material. The student must demonstrate the ability to think critically about the material and to integrate various techniques together in solving a problem. It is important for the student to have the facility to move quickly from one topic area to another.

#### **IV. Learning Activities**

In-class learning activities include lectures, discussion, group work as well as introductory training in computer packages relevant to the course. Having some or all of the classes in the computer lab enables students to gain hands on experience while in the classroom. Outside of the classroom, students are expected to do a significant amount of individual and in some cases group work to achieve learning goals. This work includes homework assignments, as well as small and large projects that give students the opportunity to practice, assimilate and expand upon the material taught in class. An important part of the learning experience will be student exposure to real problems published Interfaces, a journal of the Institute for Operations Research and Management Sciences.

# Departmental Course Syllabus MAT 317: Linear Programming

### I. Basic Information on Course and Instructor

A. <u>Purpose Statement:</u> Linear programs are mathematical problems in which linear functions are optimized subject to a set of linear constraints. These simple mathematical problems are widely used to model the world around us in areas as diverse as medicine, strategic military planning and financial portfolio management. This course provides an introduction to the algorithms, theoretical foundations, and applications of linear programming.

B. <u>Course Description</u>: This course is an introduction to the theory and applications of linear programming. Selected topics include: the simplex method for solving a linear program (LP), the geometry of LPs, variants of the simplex method, constructing mathematical models using LPs, duality theory, sensitivity analysis, integer programming, transportation and trans-shipment models, network models, program management models, and solving LPs using modern software packages (Excel, LINDO, LINGO, AMPL or MATLAB). Connections to Linear Algebra will be highlighted where relevant.

C. Course Prerequisite: MAT 205

## **II.** Learning Goals

A. <u>Content Goals</u>: Students will be proficient in the mathematical theory of linear programs and the solution techniques (simplex method and variants) available to solve linear programs. Students should also develop an appreciation and understanding of how linear programming models can be used to find optimal solutions to real-world problems. The necessity of software to solve these real-world problems will be established.

B. <u>Performance Goals:</u> Upon completing the course, students should demonstrate competence with the ideas of forming mathematical models using linear programs. Further, students should be competent at investigating solutions of these models and other linear programs using the simplex method, its variants, and computer software. Students should also be able to ascertain the robustness of a mathematical model and its results using sensitivity analysis.

## **III. Student Assessment**

A. <u>Assessment Plan:</u> Students will receive timely feedback on their homework, assignments, quizzes, student projects and examinations. Any grading policy should clearly describe the schedule for these assessment tools with the possible exception of quizzes and how they will be used to calculate grades.

B. <u>Rationale:</u> Through the use of timely feedback from homework, quizzes, student projects and examinations, students will be able to see and correct their misunderstandings and thereby improve their performance. These assessment tools are similar in manner and scope to those ways in which students will need to use their knowledge in the future. Hence these tools are appropriate to assess the accomplishment of the course's learning goals.

C. <u>Methods and Criteria</u>: We will use the assessment of homework, quizzes, student projects and examinations to evaluate student accomplishment of the course learning goals.

# **IV.** Learning Activities

A. <u>Summary of Learning Activities</u>: Learning activities will consist of a combination of lectures, group work, discussion of homework problems, and presentation of student projects. The choices and arrangements of these activities depend upon the individual instructor.

B. <u>Calendar and Outline</u>: A guide to the organization of the course and a schedule of assessment tools and quizzes will be provided to the students. Homework, quizzes and examinations will be spaced at appropriate intervals throughout the semester. It is expected that each of the major topics will be given roughly equal emphasis during the course of the semester.

c. <u>Rationale:</u> By giving students a variety of approaches to learning and doing mathematics, these activities should promote a deeper understanding of the ways in which linear programming models are used to better understand the world around us. This is certainly the major learning goal of this course. An even spacing of assessment tools insures that students get timely feedback on their progress.

Approved 11/09/11

# **Major Topics List**

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### V. Course Outline

Topics to be covered in the course are:

- i. Simplex method
- ii. Variants of the simplex method
- iii. The geometry of linear programs and the simplex method
- iv. Building LP mathematical models
- v. Duality theory
- vi. Sensitivity analysis
- vii. Integer programming
- viii. Transportation and trans-shipment models
- ix. Network models
- x. Technology and LPs: options include Excel, LINDO, LINGO, AMPL or MATLAB