Program Cover Document --- MAT 403: Advanced Calculus

I. Basic Course Information

MAT 403: Advanced Calculus is primarily a junior/senior level course. Its only prerequisite is MAT 229, but students who have had Linear Algebra, Topics in Linear Algebra, or Abstract Algebra will have an advantage. It is scheduled for two 80-minute meetings each week.

The course is devoted to the calculus of several variables, taught from the point of view of the geometry of surfaces in Euclidean space. A strong subplot is the geometry of linear maps of Euclidean space. The level of abstraction is high, but the emphasis is on examples rather than proofs.

II. Learning Goals

This course was designed as a vehicle for exposing the students to some examples encountered in research. These examples, elementary Lie groups, are applications where students can use simultaneously the calculus, algebra, and geometry they have learned.

The geometry of surfaces in Euclidean space is germane to any application of calculus in a context with more than one variable, and also has many applications within mathematics itself. The analysis in this course is a generalization of elementary calculus and its study will deepen students' understanding of basic calculus; thus the course is also highly recommended for students who expect to become teachers of calculus.

The beginning section on the topology of Euclidean space, and the use of topological notions throughout the course, will help to prepare students for more abstract examples of topologies in our Topology course. The section on topology also provides a link back to Calculus A and B, since the basic theorems of Calculus A and B are consequences of the more general topological theorems discussed here.

III. Student Assessment

This course is intended to be highly homework intensive. Weekly writing assignments will constantly provide the instructor with information on the progress of individual students. At the same time students will receive weekly feedback on their work and their progress. Tests throughout the course will provide further valuable information both for the instructor and the individual students.

Assessment of the success of Advanced Calculus in meeting its learning goals will be done through a combination of student performance in the course and in their subsequent performance in higher-level courses such as Geometry, Topology, Real Analysis, Complex Analysis, and the senior capstone seminar.

IV. Learning Activities

The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, and individual homework. Homework assignments are central to the course and have two roles. As in lower level mathematics courses, they give students practice in applying algorithms they have learned in class. But even more importantly, in higher level courses they also lead students to make discoveries and uncover examples on their own, introducing students to mathematical thought.

Departmental Course Syllabus MAT 403: Advanced Calculus

Introduction: A typical syllabus for Advanced Calculus follows this sheet. Any syllabus for Advanced Calculus should include the points listed below.

I. Basic information on course and instructor

A. Purpose statement: In this course we explore the calculus of several variables, from the point of view of surfaces in Euclidean space. The emphasis will be on examples rather than proofs.

B. Course description: We discuss the topology of Euclidean space, the derivative as a linear map, the geometry of maps whose derivatives have maximal rank, and examples including Lie groups such as Sl(2,R). The geometry of Lagrange multipliers is an optional topic.

C. Course prerequisite: MAT 229.

II. Learning goals

A. Content goals: The choice of topics covered and their emphases will be based upon the attached topics list.

B. Performance goals: Students should be familiar with the topology of Euclidean space, including the notions of continuity, compactness and connectedness. They should understand how the basic theorems of Calculus A and B are consequences of much more general theorems of topology. They should understand the geometric meaning of the derivative of a map between Euclidean spaces, and how to perform computations involving first order approximations, the chain rule, and inverse mappings. Students should be familiar with examples of surfaces of various dimensions defined both parametrically and implicitly, and know how to find their tangent planes. Students should have an elementary understanding of the Implicit function theorem, and the geometry behind it. Students should be familiar with elementary examples of Lie groups.

III. Student assessment

A. Assessment Plan: This course is intended to be highly homework intensive. Weekly writing assignments will constantly provide the instructor with information on the progress of individual students. At the same time students will receive weekly feedback on their work and their progress. Students may present solutions of homework problems to the class. Tests throughout the course will provide further valuable information both for the instructor and the individual students. A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.

B. Rationale: Through the use of regular feedback from homework and examinations, students will be able to see and correct their misunderstandings and improve their performance.

C. Methods and criteria: We will use the assessment of homework, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to use their knowledge in the future and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

A. Summary of learning activities: The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, tests and final exam.
B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework and examinations should be spaced at appropriate intervals throughout the semester.
C. Rationale: By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students have continual regular feedback on their work.

Math 403

Course Syllabus
Spring'04

Instructor: N. Hingston
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Content

1. The topology of Euclidean space R^n
   The linear structure of R^n and the inner product
   Convergence of sequences in R^n
   Open sets, closed sets, and boundaries
   Continuity
   Compactness and the Extreme Value theorem
   Connectedness and the Intermediate Value theorem

2. Derivatives of maps between Euclidean spaces
   Review of linear mappings
   The derivative matrix and first order approximation
   The chain rule

3. Maps whose derivatives have maximal rank
   The inverse function theorem and coordinate changes
   Immersions; tangent planes to immersed surfaces
   Submersions; tangent planes to level sets of mappings
   The group Sl(2,R) and the level sets of the trace function
   Constrained extreme problems and the geometry of Lagrange multipliers (time permitting)

Student Assessment

There will be homework assignments due every week. There will be two major tests during the semester and one final comprehensive exam at the end of the semester.

Grading Policy: The final grade will be determined as follows: Test 1 (30%), Test 2, (30%), Final Exam (40%). Students MUST turn in all homework assignments in order to pass the course, but the homework will not be given a numerical grade.

Learning Activities

Students are expected to participate in study groups. The study groups will be organized the first day of class. Students will be expected to meet with their study group at least for two hours every week. They should try the homework assignment of that week before meeting their peers in the study group. Students should be responsible for the preparation on that week's assignment of every member of their study group.

Students should be prepared to explain their work to the rest of the class every week.

Students are expected to bring their study groups' questions on the written assignment at the beginning of each class period.