Program Cover Sheet --- MAT 440: Mathematical Logic

I. Basic Course Information

MAT 440: Mathematical Logic is an upper level course scheduled for two 80-minute lecture periods. Sophomore standing or above is a prerequisite.

Every upper level math course is designed in part to increase a student's ability to reason logically, and this 400-level Mathematical Logic course is no different. Despite its name, however, it is not particularly designed for this goal. Instead, this course is designed to take a mathematical and philosophical view at the mechanisms of logical reasoning. There will be three major themes: the process, characteristics and limits of logical reasoning.

Much of the mathematical content and processes that will be used in this course will already have been introduced somewhere else in the curriculum, most directly in our Discrete Mathematics which serves as an introduction to techniques of proof. The most notable exception to this is that we will require some knowledge of set theory, in particular, infinite sets and cardinalities.

II. Learning Goals

The learning goals of this course reflect its major themes. Students will learn the process of logic by exploring propositional and first-order logic languages. Students will learn how to work *inside* these languages to derive proofs of statements. Secondly, students will study these languages from *outside*, in the language of metalogic. Here, students will learn the characteristics of these languages. For example, the different notions of 'truth' and 'proof' will be compared and contrasted. Lastly, students will learn the limits of mathematics in the context of completeness and consistency. It is here that students will need to learn that there are different sizes of infinity. We will take this opportunity to step away from traditional metalogic to discuss the ZF axioms of set theory; in particular the consistency and independence of the continuum hypothesis.

III. Assessment

Students will be expected to demonstrate both technical and conceptual competence in logic and metalogic. Such demonstration will be in the forms of: logical and metalogical proofs written both in and out of class, both individually and as group assignments. Also, students will need to show serious and thorough reflections on the implications of our study to the philosophical underpinnings of mathematics as a discipline. Such reflections will be in the form of periodic short essays.

IV. Learning Activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance of lectures offered by the instructor and other students, written homework assignments, oral presentations, participation in classroom discussions, and group or individual projects. These activities will address most of the general goals of the liberal arts mathematics program. In particular, students will continue learning to read, write, and explain mathematics, to think logically and abstractly, and to apply knowledge learned in previous courses to solve problems in this course.

Departmental Course Syllabus -- MAT 440: Mathematical Logic

This course will study the process, characteristics and limits of logical reasoning. To do this, we develop several logical languages from first principles. We will deduce proofs inside these languages and we will prove general theorems about these languages. Also, there will be a brief introduction to set theory to discuss infinite cardinals and their application to the study of logic.

This course is not open to freshmen and is recommended only for those with some interest and aptitude for mathematics.

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Learning Goals

Content

Students in this course will become familiar with the concept of a logical system; that is, the relative truth of statements to assumed axioms. Also, students will be able to distinguish the concept of proof with that of truth value, and will investigate whether it is possible to have a true mathematical statement for which there is no proof. They will approach this seemingly philosophical question from a rigorously mathematical standpoint. In doing so, students will become familiar with theorems on consistency and independence and develop an appreciation of Godel's Incompleteness Theorem. Lastly, students will be able to discuss the infinitely many sizes of infinite sets, developing along the way, an appreciation for the complex implications that arise out of the familiar mathematical concepts of infinity and bijective functions.

Performance

The successful logic student will be able to do all of the following:

1. Work with a specific propositional languages in which they will be able to:
    a. Calculate the truth values,
    b. Determine whether a given statement is a semantic consequence of some set of formulas,
    c. Derive proofs for simple statements,
    d. Determine whether a given statement is a syntactic consequence of some set of formulas.
2. Work with a specific first-order predicate languages in which they will be able to:
    a. Determine the scope of quantifiers and free and bound variable in statements,
    b. Determine whether a given statement is satisfied by a specific interpretation,
    c. Recognize and deduce proofs of simple statements.
3. Prove that there are infinitely many sizes of infinity including defining all relevant terms.
4. Prove that no logical system that includes the integers is complete including all relevant terms.

Assessment

Students will be assessed through graded homework assignments, oral communication, and in class and final examinations. The homework assignments and examinations will directly assess the learning listed above.

Homework assignments will provide students with opportunities to attempt lengthier, more challenging problems than is possible on an examination as well as offering students practice at exam-style problems. Moreover, written assignments will give students an opportunity to reflect on the philosophical implications of the course content. Examinations, which normally preclude both the use of books and the practice of group discussion, enable the professor to assess the knowledge an individual student has readily available.

Learning Activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance of lectures offered by the instructor and other students, written homework assignments, oral presentations, participation in classroom discussions, and group or individual projects.

Mathematical Logic

Syllabus

Course: Math 440; section 1
Time: Tuesdays and Fridays 3:30-4:50
Room: SCP 230

Instructor: Andrew Clifford
Office: SCP 206
Phone: 771-3060
E-mail: cliffan@tcnj.edu
Office Hours: Tuesdays 11:00 - 12:30 and Wednesdays 9:30 – 11:00.
In addition, I will be available other times by appointment.

Course Description: Every upper level math course is designed in part to increase a student's ability to reason logically, and this 400-level Mathematical Logic course is no different. Despite its name, however, it is not particularly designed for this goal. Instead, this course is designed to take a mathematical and philosophical view at the mechanisms of logical reasoning. There will be three major themes: the process, characteristics and limits of logical reasoning.

We will study various types logical languages. We will learn to work inside these languages, deriving proofs, etc. and we will look at these languages from the outside to learn their global properties. We will also take opportunities to take philosophical and mathematical tangents where we will explore the implications of our study. Our principal text will be Metalogic: An Introduction to the Metatheory of Standard First Order Logic, by Geoffrey Hunter. However, further resources will be made available during the course.

Resources: SOCS is TCNJ's on-line courseware system. I will put resources for the course on SOCS. To access those resources, go to http://socs.tcnj.edu and log in. Your login information is the same as that for your UNIX account (usually some version of your last name with a number, e.g. clifford2

Learning Goals: Students will learn the process of logic by exploring propositional and first-order logic languages. Students will learn how to work *inside* these languages to derive proofs of statements. Secondly, students will study these languages from *outside*, in the language of metalogic. Here, students will learn the characteristics of these languages. For example, the different notions of 'truth' and 'proof' will be compared and contrasted. Lastly, students will learn the limits of mathematics in the context of completeness and consistency. It is here that students will need to learn that there are different sizes of infinity. We will take this opportunity to step away from traditional metalogic to discuss the ZF axioms of set theory; in particular the consistency and independence of the continuum hypothesis.

Assessment: Students will be expected to demonstrate both technical and conceptual competence in logic and metalogic. Such demonstration will be in the forms of: logical and metalogical proofs written both in and out of class, both individually and as group assignments. Also, students will need to show serious and thorough reflections on the implications of our study to the philosophical underpinnings of mathematics as a discipline. Such reflections will be in the form of periodic short essays.

Grading:

Two in-class tests, 10/10 and 11/21, 50%
Written assignments, due every other Friday, 20%
Final Exam, during Finals week, 30%

I will adjust your grade according to the more nebulous facets of student performance. [i.e. class participation and attendance] and to the challenge problems.

Course Outline:

The course will begin with a two week introduction to cardinality. The principle result that we will study will be Cantor's Theorem on Infinite Sets.

Following this we will spend about five weeks on truth functional propositional logic. This will involve the development of a specific propositional logic. The principle topics will be: the Interpolation Theorem, consistency, the Deduction Theorem, and completeness.

Next, we will spend about 3 weeks on first order predicate logic. As before, we will develop a predicate logic. Our principle topics will be: models, proofs, consistency, and model isomorphisms.

The balance of the semester will be spent on ZF-Set theory with an emphasis on the axiom of choice and the continuum hypothesis.