MAT 265/Introduction to Financial Mathematics
Program Cover Document

I. Basic Course Information

Undergraduate Bulletin course description: "An introduction to mathematical and numerical models used to price financial securities and make risk estimates. Topics include time value of money, annuities and cash flows, loans, bonds, general cash flows and portfolios, immunization, derivatives, options, swaps, and hedging and investment strategies."

MAT 265 is a course for students interested in understanding how mathematical models are used in financial and actuarial work. The course is graded and will meet for two eighty-minute class periods per week.

II. Learning goals

MAT 265: Introduction to Financial Mathematics will provide students an introduction to the mathematical and numerical models used to price financial securities and to make risk estimates. The field of mathematical finance is a modern subfield of probability, a theory of mathematics created by Pascal, Bernoulli, and Laplace for making educated decisions in the face of uncertainty. Financial mathematics is the specific application of these methods to make decisions in the face of uncertainty in the financial markets. This field is central to the development of modern financial instruments in the economic markets. A central example is the 1973 Black-Scholes model for financial derivatives, for which Black and Scholes were awarded the 1997 Nobel Prize in Economics.

The primary learning goals of the course are to 1) introduce students to the concepts in financial mathematics; 2) introduce students to financial instruments as they relate to financial mathematics 3) introduce students to the use of mathematical models for financial products; 4) develop student abilities to create and apply mathematical models. The specific content goals contained in items (1), (2) are: the concepts of fixed income, equities, and financial derivative products; the time value of money; compound interest; annuities; cash flows; loan concepts and amortization; mathematics of fixed income products; portfolios; and immunization.

The Department's three primary learning goals are:

1. Students should be able to effectively communicate mathematical and/or statistical ideas to diverse audiences, both orally and in writing
2. Students should be effective problem solvers, using technology and connections between different areas of disciplinary knowledge as appropriate
3. Students should demonstrate engagement in their discipline.

Department learning goal (1) on developing written communication skills will be met through the regular assignment of problem sets. Department learning goal (2) will be met through the skills developed to solve mathematical problems in finance and to use computer software to generate and analyze financial models and tables. Department leaning goal (3) will be met by showing students the interplay between mathematical theory and its applications in the field of financial studies.

MAT 265 should appeal to students interested in pursuing advanced studies and careers in finance, financial mathematics, and actuarial science. The MAT 265 course will provide these students with both theoretical knowledge, and experience with actual case studies of applications of mathematical finance. The course is an important component of the minor in Actuarial and Financial Risk Studies, which is housed in the Department of Mathematics and Statistics.

MAT 265 will also provide students the knowledge to pass the FM/2 Financial Mathematics exam, the second exam administered by the national actuarial organizations. Passage of this exam is helpful for students seeking to obtain internships and pursue careers in actuarial science and finance.

III. Student assessment

Student work will be the primary measure to assess how well students meet the learning goals of MAT 265. A combination of homework sets, quizzes, and tests throughout the course will be given to assess student progress. A secondary measure will be the success rate of students taking the FM/2: Financial Mathematics examination.

IV. Learning activities

The specific choices of learning activities will depend upon the instructor. It is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, computer assignments, quizzes, tests and final exam.

Suggested Syllabus for Course Instructors

MAT 265: Introduction to Financial Mathematics

Class Meetings: Two 80-minute class meetings per week.

Prerequisites: MAT 128: Calculus II is the formal prerequisite. An introductory knowledge of probability is also assumed.

Course Description/Learning Goals: This course introduces the students to the concepts in financial mathematics, a field of mathematics that uses mathematical and numerical models to make educated decisions in the face of uncertainty in the financial markets. The primary learning goals of the course are to:

1. Introduce the concepts of financial mathematics;
2. Provide an introduction to financial instruments related to financial mathematics;
3. Introduce students to the use of mathematical models for financial products;
4. Develop student abilities to create, derive, and apply mathematical models.

The specific course content goals are to introduce the concepts of fixed income, equities, and financial derivative products, and make the connections between practical applications on Wall Street today with applied mathematics and problem solving real-life financial problems and case studies. Topics covered include extensive work with the time value of money, annuities, and uneven cash flows; loan concepts and amortization, bond (fixed income product) vocabulary and math, general cash flows and portfolios, and immunization. Some concepts from probability related to financial mathematics will also be covered. A more extensive topics list can be found at the end of the syllabus.

Purpose of the Course: MAT 265 will provide students with a sound foundation in both the theoretical and pragmatics aspects of financial mathematics. The course provides an overview of how theoretical concepts and models are used in real-life applications. Students who desire to enter the actuarial or finance professions will be given a solid theoretical understanding of the essential topics in these fields. In addition, this course will cover all of the required topics found on the actuarial FM/2:Financial Mathematics examination.

Fourth hour: All TCNJ courses have an associated fourth hour meeting time. The "fourth hour" of this course is devoted primarily to acquisition of technology skills. Specifically, students will learn how to use software programs such as Microsoft Excel to calculate financial tables such as duration of bonds and amortization schedule. This time will also be devoted to working examples of problems from the course.

Required text: Financial Mathematics: A Practical Guide for Actuaries and Other Business Professionals. Chris Ruckman and Joe Francis. Publisher: BPP Professional Education; 2nd edition (October 2005). ISBN: 0975313649

Recommended texts (will be on reserve):

1. ACTEX Study Manual for the SOA Exam FM and CAS Exam 2, Samuel A. Broverman, ACTEX Publications, ISBN 978-1-56698-745-5, http://www.actexmadriver.com/ .
2. Derivatives Markets (3rd Edition) (Pearson Series in Finance) 3rd Edition by Robert L. McDonald (Author) Publisher: Prentice Hall; 3 edition (September 6, 2012) ISBN: 0321543084

About the Actuarial Exam on "Financial Mathematics:" The central clearinghouse for up-to-date information on Actuarial Careers and Exams is < http://www.BeAnActuary.org/ >. The Financial Mathematics Exam is a three-hour multiple choice examination and is referred to as Exam FM by the SOA and Exam 2 by the CAS. The examination is jointly sponsored and administered by the SOA, CAS, and the Canadian Institute of Actuaries (CIA). The examination is also jointly sponsored by the American Academy of Actuaries (AAA) and the Conference of Consulting Actuaries (CCA). The Financial Mathematics Exam is administered as a computer-based test. For additional details, Please refer to "Computer- Based Testing Rules and Procedures".

What is an Actuary? An actuary is a business professional who analyzes the financial consequences of risk. Actuaries use mathematics, statistics, and financial theory to study uncertain future events, especially those of concern to insurance and pension programs. Actuaries may work for insurance companies, consulting firms, government, employee benefits departments of large corporations, hospitals, banks and investment firms, or, more generally, in businesses that need to manage financial risk. A career as an Actuary is better described as a "business" career with a mathematical basis than as a "technical" mathematical career.

Course Topics: The course will cover foundational topics in financial and actuarial mathematics. In particular, the topics will include those on the Financial Mathematics exam given by the Society of Actuaries and Casualty Actuarial Society (see Appendix), as well as additional topics. These additional topics will include financial derivatives, the inputs to the Black-Scholes equation, Value at Risk (VAR), and the theory underlying cash flows and the derivations of calculations such as portfolio yield rate and the time-weighted rate of return.

Expectations: All students are expected to…

- Attend class regularly and participate in in-class activities.
- Read the portions of the text as assigned.
- Make serious attempts at all of the assigned weekly problem sets and in preparation for exams.
- Use the resource of their fellow students, the tutoring center, and their instructor to seek answers to questions that arise in class, in the readings, and on the homework

Required Class Materials

Calculator and Computer: Each student is required to bring an approved calculator to every class session. Extensive use of this calculator, approved for use on the Actuarial exam, will be covered in class. The TI BA-II + (BA = Business Analyst) is the STRONGLY RECOMMENDED calculator.

Due to exam regulations, no graphing calculators or cell phone calculators will be allowed, even if they have full financial calculator capabilities. Texas Instruments has a nice app that replicates the calculator, but it is absolutely not allowed on exams in class or on the Actuarial exam.

Classroom Policies

Attendance: ***All students are expected to attend all classes***. It is assumed that any information given out during class has been delivered to all students. A student who is absent for a test **will not** be permitted to make up the test unless some arrangement has been made with me in advance. Approval for missing a test will be rare and based on truly exceptional circumstances. In the case of illness, a doctor's note will be required. Please view TCNJ's attendance policy:

http://www.tcnj.edu/~recreg/policies/attendance.html

Academic Honesty: Please make sure you are familiar with TCNJ's academic honesty policy. Any suspected violation of this policy will be confronted in strict accordance with the policy. **http://www.tcnj.edu/~academic/policy/integrity.html**

Students with Disabilities: See TCNJ's Americans with Disabilities Act (ADA) policy available on the web: http://www.tcnj.edu/~affirm/ada.html .

Graded Assignments

Exams & Quizzes
10% of your grade will be based on a calculator quiz given in week 3.
20% and 30% of your grade will be based on two subsequent exams, the worst performing exam of the two will be worth 20%

Problem Sets & Class Participation
10% of your grade will be based on successful completion of the assigned problem sets.

Final Exam (30%): The final exam for this course will be a comprehensive exam. See TCNJs Final Exam policy at http://www.tcnj.edu/~academic/policy/finalevaluations.htm

Extra Credit. My grading policy is generous and is designed for students to succeed. There will be absolutely no extra credit issued or assigned.

Suggested Weekly Schedule of Topics

Introduction to Financial Mathematics

Week 1: Intro to course. Calculator use. Interest. Yield terms Nominal, Effective, Periodic.

Week 2: Inflation. Investment return. Uneven cash flows. IRR and NPV *Problem Set #1 due*

Week 3: ROI, Yield on investment. Annuities. Annuity Due. Perpetuities. *Calculator Quiz*

Week 4: Intro to equities. Dividend discount models. Stock valuation. Intro to asset allocation concepts. Review for test #1. *Problem Set #2 due.*

Week 5: Margin and margin calculations. Installment plans. Amortization plans.

Week 6: *Test 1*. Loan amortization. Constructing amortization tables (computer lab). Value at Risk.

Week 7: Intro to fixed income products. Reading bond quotes. Bond vocabulary. Calculating bond yield. Discounts and premiums. *Amortization table due (PS #3).*

Week 8: Sinking funds. Embedded options (on bonds). Preferred stock and convertibility. *Problem Set #4 due.*

Week 9: Term structure. Duration and convexity. Calculating duration and convexity (computer lab). Review for test #2. *Problem Set #5 due.*

Week 10: *Test 2.* Introducing financial derivative products: options, futures, forwards, swaps.

Week 11: Option/derivative math: break even, ITM, OTM, ATM. Net debits and net credits. Swap terms, conventions, and uses. *Problem Set #6 due.*

Week 12: Option Strategies. Bull/Bear Call/Put Spreads. Straddles. Strangles. Synthetics relations. Futures. Forwards. Swaps.

Week 13: Immunization. Constructing hedged portfolios. Leverage. *Problem Set #7 due.*

Week 14: Review for final exam and review for FM/2 exam. Next steps.

Appendix: Topics Covered on the SoA/CAS Financial Mathematics Exam

(The percentages indicate the percentage of the exam devoted to that topic.)

1. Interest Theory (65-80%)
    a. Time Value of Money (5-15%)
        i. The candidate will be able to define and recognize the definitions of the following terms: interest rate (rate of interest), simple interest, compound interest, accumulation function, future value, current value, present value, net present value, discount factor, discount rate (rate of discount), convertible m-thly, nominal rate, effective rate, inflation and real rate of interest, force of interest, equation of value
        ii. The candidate will be able to:
            1. Given any three of interest rate, period of time, present value, current value, and future value, calculate the remaining item using simple or compound interest. Solve time value of money equations involving variable force of interest.
            2. Given any one of the effective interest rate, the nominal interest rate convertible m- thly, the effective discount rate, the nominal discount rate convertible m-thly, or the force of interest, calculate any of the other items.
            3. Write the equation of value given a set of cash flows and an interest rate.
    b. Annuities/cash flows with payments that are not contingent (5-20%)
        i. The candidate will be able to define and recognize the definitions of the following terms: annuity-immediate, annuity due, perpetuity, payable m-thly or payable continuously, level payment annuity, arithmetic increasing/decreasing annuity, geometric increasing/decreasing annuity, term of annuity
        ii. For each of the following types of annuity/cash flows, given sufficient information of immediate or due, present value, future value, current value, interest rate/yield rate, payment amount, and term of annuity, the candidate will be able to calculate any remaining item
            1. Level annuity, finite term
            2. Level perpetuity
            3. Non-level annuities/cash flows
                a. Arithmetic progression, finite term
                b. Arithmetic progression, perpetuity
                c. Geometric progression, finite term
                d. Geometric progression, perpetuity
                e. Other cash flows
    c. Loans (5-20%)
        i. The candidate will be able to define and recognize the definitions of the following terms: principal, interest, term of loan, outstanding balance, final payment (drop payment, balloon payment), amortization, sinking fund
        ii. The candidate will be able to:
            1. Given any four of term of loan, interest rate, payment amount, payment period, principal, calculate the remaining item.
            2. Calculate the outstanding balance at any point in time.
            3. Calculate the amount of interest and principal repayment in a given payment.

4. Given the quantities, except one, in a sinking fund arrangement calculate the missing quantity.
5. Perform similar calculations to 1-4 when refinancing is involved.

d. Bonds (5-20%)
   i. The candidate will be able to define and recognize the definitions of the following terms: price, book value, amortization of premium, accumulation of discount, redemption value, par value/face value, yield rate, coupon, coupon rate, term of bond, callable/non- callable
   ii. Given sufficient partial information about the items listed below, the candidate will be able to calculate the any of the remaining items.
      1. Price, book value, amortization of premium, accumulation of discount
      2. Redemption value, face value
      3. Yield rate
      4. Coupon, Coupon rate
      5. Term of bond, point in time that a bond has a given book value, amortization of premium, or accumulation of discount

e. General Cash Flows and Portfolios (5-20%)
   i. The candidate will be able to define and recognize the definitions of the following terms: yield rate/rate of return, dollar-weighted rate of return, time-weighted rate of return, current value, duration (Macaulay and modified), convexity (Macaulay and modified), portfolio, spot rate, forward rate, yield curve, stock price, stock dividend
   ii. The candidate will be able to:
      1. Calculate the dollar-weighted and time-weighted rate of return
      2. Calculate the duration and convexity of a set of cash flows.
      3. Calculate either Macaulay or modified duration given the other.
      4. Use duration and convexity to approximate the change in present value due to a change in interest rate
      5. Calculate the price of a stock using the dividend discount model

f. Immunization (5-15%)
   i. The candidate will be able to define, derive, recognize and the definitions of the following terms: cash flow matching, immunization (including full immunization), Redington immunization
   ii. The candidate will be able to:
      1. Construct an investment portfolio to fully immunize a set of liability cash flows.
      2. Construct an investment portfolio to match present value and duration of a set of liability cash flows
      3. Construct an investment portfolio to exactly match a set of liability cash flow

2. Financial Economics (20-35%)
   a. General Derivatives (0-5%)
      i. The candidate will be able to define and recognize the definitions of the following terms: derivative, underlying asset, over the counter market, short selling, short position, long position, ask price, bid price, bid-ask spread, lease rate, stock index, spot price, net profit, payoff, credit risk, dividends, margin, maintenance margin, margin call, mark to market, no-arbitrage, risk-averse
      ii. The candidate will be able to evaluate an investor's margin position based on changes in asset values

b. Options (0-5%)
    i. The candidate will be able to define and recognize the definitions of the following terms: call option, put option, expiration, expiration date, strike price/exercise price, European option, American option, Bermudan option, option writer, in-the-money, at-the-money, out-of-the-money, covered call, naked writing, put-call parity
    ii. The candidate will be able to evaluate the payoff and profit of basic derivative contracts.
c. Forwards and Futures (0-10%)
    i. The candidate will be able to define and recognize the definitions of the following terms: forward contract, futures contract, outright purchase, fully leveraged purchase, prepaid forward contract, cost of carry.
    ii. The candidate will be able to:
        1. Determine forward price from prepaid forward price.
        2. Explain the relationship between forward price and futures price.
        3. Explain the relationship between forward price and future stock price.
        4. Use the concept of no-arbitrage to determine the theoretical value of futures and forwards
        5. Given sufficient partial information about call premium, put premium, forward price, strike price and interest rate, calculate any remaining item using the put-call parity formula
d. Swaps (0-5%)
    i. The candidate will be able to define and recognize the definitions of the following terms: swap, swap term, prepaid swap, notional amount, market value of a swap, swap spread, deferred swap, simple commodity swap, interest rate swap, interest rate swap net payments.
    ii. The candidate will be able to use the concept of no-arbitrage to determine the theoretical values of swaps.
e. Hedging and Investment Strategies (5-15%)
    i. The candidate will be able to define and recognize the definitions of the following terms: hedging, arbitrage, diversifiable risk, non-diversifiable risk, spreads (option, bull, bear, vertical, box, ratio), collar width, collared stock, zero-cost collar, straddle, strangle, written straddle, butterfly
    ii. The candidate will be able to:
        1. Explain how derivative securities can be used as tools to manage financial risk.
        2. Explain the reasons to hedge and not to hedge.
        3. Evaluate the payoff and profit of hedging strategies.