Departmental Course Syllabus --- MAT 301: Number Theory

Introduction: A typical syllabus for Number Theory follows this sheet. Any syllabus for Number Theory should include the points listed below (the required course requirement sections).

I. Basic information on course and instructor

A. Purpose statement: Number Theory introduces students to some of the classical problems in elementary number theory. While learning the subject matter of number theory, the course will also develop a student's ability to reason abstractly, to read mathematics, and to prove theorems. Through the use of challenging problems, the course should also develop a student's problem solving ability and introduce students to the joys of mathematics. The course should also provide a firm foundation in modular arithmetic for students taking Abstract Algebra.

B. Course description: An introduction to number theory. Topics include divisibility, primes, unique factorization, Diophantine equations, congruences, and quadratic reciprocity. Optional topics may include sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves.

C. Course prerequisites: MAT 200.

II. Learning goals

A. Content goals: Students will gain acquaintance with many basic topics in elementary number theory. Students will learn about primes, unique factorization, congruences, divisibility, Diophantine equations, primitive roots, and quadratic reciprocity. Other optional topics such as sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves may be included at an instructor's discretion.

B. Performance goals: At the completion of the course, students should demonstrate competence with number theory concepts. A successful number theory student should be able to do modular arithmetic and know how to use modular arithmetic to study Diophantine equations. They should know basic procedures to determine whether or not a Diophantine equation has solutions. They should understand the definition of a prime, their central role in arithmetic problems and be knowledgeable about primes in number systems other than the integers. They should also be proficient at finding and working with primitive roots. Students should possess improved reasoning and proof-writing ability and should exhibit a more mature ability with proofs than would normally be expected of a student at the completion of MAT 205. In particular, they should be comfortable with important techniques of proofs such as Fermat's theory of descent.

III. Student assessment

A. Assessment plan: Students will receive regular feedback on their work through the assignment of homework, quizzes, student presentations and examinations. A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.

B. Rationale: Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.

C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to use their knowledge in the future of and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

A. Summary of learning activities: Learning activities will consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend

upon the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals. Students should be expected to use appropriate tools, including computers and simple computer programs, as well as concrete models or algorithms, for explorations in Number Theory.

B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester. As a general rule, it is expected that each of the major topics of primes, unique factorization, congruences, divisibility, Diophantine equations, primitive roots, and quadratic reciprocity will be given equal emphasis during the course of the semester.

C. Rationale By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper number theory understanding and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students continual regular feedback on their work.

MAT 301 - Number Theory

Spring 2015

Section 01
M/Th: 2:00pm - 3:20pm -- SCP 229
W: 4:00pm - 4:50pm -- SCP 201/SCP 224

Section 02
M/Th: 3:30pm - 4:50pm -- SCP 229
W: 4:00pm - 4:50pm -- SCP 201/SCP 224

Instructor: Steffen Marcus
Office: SCP 245
Email: marcuss@tcnj.edu
Office Hours:
M: 10:00am - 11:00am
W: 3:00pm - 4:00pm

Course Description:
MAT 301/Number Theory
1 course unit
Prerequisite: MAT 200
This course is an introduction to number theory. Topics include divisibility, primes, unique factorization, Diophantine equations, congruences, and quadratic reciprocity. Optional topics may include sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves. This course includes a weekly lab component where we will be writing and executing cryptographic computer algorithms using the modular arithmetic techniques studied in lecture. Please refer to the departmental program cover sheet and syllabus for this course.

Website: All course information, announcements, and grades will be posted on this Canvas portal website. It also contains information that may help you succeed in this course. I'll assume you are keeping up to date with its contents.

Course Materials and Text: Strayer, *Elementary Number Theory*, ISBN 9781577662242

Course Purpose and Learning Goals: The main goal of the course is to provide an introduction to elementary and contemporary number theory. It will include many examples of intermediate and advanced techniques, methods of proofs, and rigorous analytic thinking used in upper level mathematics. In particular, the hope is that you will learn how to use your understanding of mathematics and maturity in proof writing to study a variety of interesting mathematical topics, including:

- divisibility,
- quadratic reciprocity,
- Diophantine equations,
- modular arithmetic,
- cryptography,

- various other advanced topics.

MAT 301 provides a good foundation in mathematical thinking for other advanced mathematics or computer science courses.

Course Requirements: The course will involve a combination of lecture, group work, computer labs and in class activities. Graded components include highlighted problems, quizzes, in class projects, two in class tests, lab work, and a final exam. I will be also assigning reading from the textbook and further

recommended problems for you to think about. Mathematics is learned by consistently reinforcing the material. I will be operating under the assumption that you are reading the textbook at the pace suggested by the reading assignments, and working on the recommended problems throughout the term and not just the day before the exam.

Evaluation:
20% quizzes
10% in class assignments, highlighted problems and lab work
20% Test 1
20% Test 2
30% final exam
I reserve the right to change the grading scheme as I see fit. Any other grading scheme I use will only be
beneficial to your grade as compared to the above standard. This course will be graded using the
college's standard letter grade scale. Please refer to the college's policies regarding final exam,
evaluation, and reading days.

Attendance: I will operate under the assumption that you are attending all lectures and lab sections. I certainly think it is a really good idea to do so. My feelings will never be hurt, however, if you happen to miss a lecture. Even in the case of an emergency, you will always be responsible for the material presented in a class you may have missed. The only way to hand in homework and other written assignments is by attending class when they are due. The only way to take a quiz, test or final exam is to attend class as they are administered. Please be in touch with me immediately or as soon as possible regarding any extenuating circumstances you might have. Please also refer to the college's policies regarding absence and attendance.

Collaboration: I never recommend doing mathematics alone. Part of the reason I love math is that it can be such a social discipline if you let it. Please study together. Work together on the recommended problems. Teach someone something you understand - it's the best way to reinforce it. That being said, when it comes time to write up any highlighted problems, *you must work alone*. The expectation is that when you are working with other people, you are talking things out and perhaps writing stuff on a blackboard or scrap piece of paper, but you are not taking personal notes. When you go back home and write your homework up in good, your work should be in your own words and not copied. You may also consult the college's policies regarding academic integrity.

Due Dates:
Quizzes: Monday Feb 9/Feb 23/Mar 30/Apr 13; Thursday May 7

Test 1: Monday, March 9, 2015 in class.
Test 2: Monday, April 27, 2015 in class.
Final Exam: TBA (arranged by university administration)

This syllabus is intended to give the student guidance in what may be covered during the term and will be followed as closely as possible. However, I reserve the right to modify, supplement and make changes as the course needs arise.

Finally, I'll refer you to the college's policies on the Americans with Disabilities Act.