Program Cover Document - MAT 331: Numerical Analysis

I. Basic Course Information

Numerical Analysis focuses on developing, utilizing and comparing methods to approximate solutions to mathematical problems. This course will particularly focus on problems that arise in Calculus and Linear Algebra, but cannot be solved using the techniques of those courses. Beyond being able to execute the techniques, a theoretical understanding of algorithm error and efficiency will be established. This course has three major themes: approximating solutions to mathematical problems that cannot be solved exactly, analyzing the errors in these approximations, and understanding the efficiency of an algorithm used to arrive at an approximation. Proficiency in numerical methods requires a mix of theory to prove properties of numerical algorithms, and calculations which require implementing and programming algorithms.

Course prerequisites: MAT 128: Calculus B, and MAT 205: Linear Algebra.

II. Learning Goals

The primary goal of this course is for students to appreciate numerical analysis as a field in which there is not a concrete notion of "the right answer". Instead, students should come to understand that numerical analysis requires us to construct a reasonable approximation and to use mathematical techniques to analyze the accuracy and performance of that approximation.

By the end of the course students will be able to:

1. Derive from first principles methods for approximating the solution to problems that arise from calculus and linear algebra.
2. Understand the important subject of error that arises in scientific computing, as well as analyzing methods for detecting, predicting and controlling error.
3. Implement and execute numerical methods on a computer.
4. Utilize proof techniques from analysis and linear algebra to study properties of numerical methods.

III. Learning Activities

Learning activities may consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend on the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals.

IV. Student Assessment

Students will receive feedback on their work through either homework assignments, projects, or examinations.

V. List of Major Course Topics

The following list of topics will be covered in the course. Items denoted with ** are optional, *however it is expected that the instructor will choose at least two of these topics for inclusion in the course.*

1. Error Analysis and Computer Representation of Numbers
2. Roots of Nonlinear Equations
    a. Bisection Method including Error Analysis
    b. Newton's Method
    c. Secant Method **
    d. Fixed Point Methods **
3. Polynomial Interpolation
    a. Lagrange Polynomials and Error Formula
    b. Divided Differences and Newton's Interpolation
    c. Splines **
4. Numerical Differentiation and Integration
    a. Numerical Differentiation – Forward, Backward, and Center methods with Error Analysis
    b. Higher Derivatives**
    c. Trapezoidal Rule and Simpson's Rule
    d. Richardson Extrapolation and Romberg Integration **
    e. Gaussian Quadrature**
5. Numerical Linear Algebra
    a. Gaussian Elimination
    b. LU factorization and operation count
    c. Pivoting Strategies **
    d. Jacobi and Gauss-Seidel Iterative Methods
    e. Power Method
    f. Conjugate Gradient Method **

COURSE DESCRIPTION
MAT 331/Numerical Analysis
Numerical Analysis is a course which focuses on methods of approximating solutions to problems for which the techniques of the earlier Calculus and Linear Algebra courses fail. A course covering methods of approximation, errors in approximation, and efficiency of algorithms. (1 course unit)
Prerequisites: MAT 128 and MAT 205

INSTRUCTOR INFORMATION

Instructor: Dr. Jana Gevertz
Email: gevertz@tcnj.edu
Office: Science Complex P212
Phone: 609-771-3314
Office Hours: Monday 8:45-9:45am, Wednesday 4:30-6:00pm, and by appointment

COURSE INFORMATION

Time: Monday and Thursday, 10:00-11:20
Location: Science Complex P223
Fourth hour: Students will be expected to work on in-depth projects throughout the semester that will use computational resources to investigate theory and techniques that are introduced during lecture.
Course Website: This course utilizes the Canvas course management system. General course material, homework assignments, project details, and any other important documents will be posted on Canvas.

COURSE MATERIALS

Required Textbook: *Numerical Mathematics and Computing*, 7th Ed. by Cheney & Kincaid
Supplemental Books (on reserve in Library):

1) *Numerical Analysis* by Burden and Faires
2) *Introduction to Numerical Methods and Analysis* by Epperson
3) *Elementary Numerical Analysis* by Atkinson and Han

Technology: Graphing calculator (without symbolic math capabilities) and MATLAB. MATLAB is freely available for download onto your personal computer at http://helpdesk.pages.tcnj.edu/campus-software/home-use-software/
Notebook recommendation: I highly recommend you use a three-ring *binder* instead of a notebook for the course, as handouts will regularly be given to supplement the lecture.

COURSE PURPOSE & LEARNING GOALS:

Much of science today involves complex computations carried out via mathematical algorithms. Individuals who utilize computer software packages and execute numerical methods often have little knowledge about the mathematical underpinnings of these algorithms. Through the study of numerical analysis, you will be better able to understand how mathematical algorithms work, and how these algorithms can fail. Further, you will be better prepared to evaluate the accuracy of a computer's results.

This course has three major themes: approximating solutions to mathematical problems that cannot be solved exactly, analyzing the errors in these approximations, and understanding the efficiency of an algorithm used to arrive at an approximation. Proficiency in numerical methods requires a mix of theory to prove properties of numerical algorithms, and calculations which require implementing and programming algorithms.

By the end of the course students will be able to:

1. Obtain an intuitive and working understanding of where some numerical methods for solving mathematics problems come from, and why they work.
2. Understand the important subject of error that arises in scientific computing, as well as learning methods for detecting, predicting and controlling error.
3. Implement and execute numerical methods on a computer.
4. Understand how central results from calculus play an important role in numerical analysis.
5. Appreciate numerical mathematics as a field in which there is not a concrete notion of "the right answer". Instead, students should come to appreciate that numerical mathematics is a field that requires us to construct a reasonable approximation and to be able to think clearly and logically enough to analyze the accuracy and performance of that approximation.

COURSE REQUIREMENTS AND GRADING

You will be assigned readings from your textbook and supplemental references that are held on reserve in the library. To assess your understanding of the course material, homework assignments (often requiring MATLAB) will be given every other week. You can also anticipate two group projects, two in-class exams, and a final exam. The following rubric will be used to determine each student's final grade. In addition, very good/poor class participation and attendance can affect your final grade by one-third of a letter grade (up/down). The instructor reserves the right to change or modify this formula as needed.

Homework Assignments/Quizzes	20%
Projects Tentative Due Date for Project #1: Monday 10/7 Tentative Due Date for Project #2: Thursday 11/21	15%
Exam #1 Tentative Date: Monday 10/14	20%
Exam #2 Tentative Date: Monday 11/25	20%
Final Exam Date TBA	25%

Homework: Homework will be assigned approximately every other week. Assignments will be announced in class and posted on Canvas. You are responsible for knowing all due dates whether you are in class the day an assignment is given or not. You are encouraged to work on the homework with your classmates, although your final write-up must be your own. Homework will consist of a mix of problems: some can be solved by hand, whereas others will require the use of a computer. Announced quizzes can also be given separate from the homework.

Attendance, Lateness and Make-ups: Homework and exams are based on material presented in class, so attendance is integral to learning the course material. Therefore, I expect you to come to class and participate regularly in class discussion. Assignments are due at the beginning of class. Make-up exams will only be given in extraordinary circumstances when written documentation of the emergency is provided to me. Details on TCNJs College Attendance Policy can be found at: http://www.tcnj.edu/~recreg/policies/attendance.html.

Academic integrity: You are expected to know the college's policy on academic integrity, which can be found at http://www.tcnj.edu/~academic/policty/integrity.html. While I encourage you to work with your classmates on assignments, each write-up must represent your own work. Needless to say, cheating on exams in any form will not be tolerated. Other forms of academic integrity violations include finding solutions to homework problems online or in a solution manual and submitting the solution as your own (without reference), and using MATLAB code written by a classmate. These violations are not to be taken lightly, so please refrain from ever representing others work as your own.

Students in need of accommodations: Students with documented needs for in-class accommodations should make me aware of this AS SOON AS POSSIBLE! All documented accommodations will be respected, as specified by the Americans with Disabilities Act Policy (http://www.tcnj.edu/~affirm/ada.html).

TENTATIVE COURSE SCHEDULE (Subject to Change)

Introduction and Preliminary Mathematics (5 classes)

Introduction to numerical mathematics; algorithms; error; Taylor's Theorem; computer programming; computer representation of numbers

Roots of Nonlinear Equations (3 classes)

Bisection method; Newton's method; Secant method (optional); convergence analysis

Polynomial Interpolation (2-3 classes)

Lagrange interpolating polynomial; Lagrange polynomial error formula; Newton's interpolation and divided differences (optional); Hermite interpolation (optional)

Splines (3-4 classes)

Lower degree splines; cubic splines; parametric curves; applications

Numerical Differentiation (1-2 classes)

Forward, backwards and centered methods; error analysis; interpolation; higher derivatives; method of undetermined coefficients

Numerical Integration (3-4 classes)

Basic quadrature; trapezoidal rule; Simpson's rule; Romberg integration; Gaussian quadrature; adaptive quadrature (optional)

Numerical Linear Algebra (5-6 classes)

Gaussian elimination; pivoting strategies; operation counts; matrix factorizations; eigenvalues; power method; iteration methods; Newton's method for systems (optional)