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# Analysis of a Survey on Power-Based Pers

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### Introduction:

Power-based personal violence is a form of harm resulting from an imbalance of power in personal relationships. The University of Tennessee defines power-based violence as "a type of violence committed by an offender who uses the assertion of power, control, and/or intimidation in order to harm another." Five dimensions of violence make up power-based personal violence: emotional abuse, physical abuse, controlling behavior, sexual assault, and stalking.

This survey was administered in the 2010 Spring semester. The survey covered a wide range of topics: the questions that deal with the five dimensions of personal violence. Power-based personal violence is a form of harm resulting from an imbalance of power in personal relationships. The University of Tennessee defines power-based violence as "a type of violence committed by an offender who uses the assertion of power, control, and/or intimidation in order to harm another." Five dimensions of violence make up power-based personal violence: emotional abuse, physical abuse, controlling behavior, sexual assault, and stalking.

### Methods:

An effective clustering method first needed to be selected and used. K-means clustering is an old and versatile method for clustering. The user specifies the number of centers/means or clusters. There are many ways of doing this. The Calinski criterion is often used, but this time it gave a result that would yield very interesting results. Another oft used method for selecting the number of clusters is the within-group sum of squares (WSS). Sum of squares is just the sum of the squared Euclidian distances from each point to the mean of the group. As the number of clusters increases, this measure decreases. The optimal number of clusters lies before there are very small reductions in WSS for each additional cluster. So, we're looking just after the elbow (see Figure II).

K-means clustering only has one major assumption: the data should all have the same scale. The data was quite limiting because it was so skewed (see Table II). K-means clustering was used with seven centers. The clusters were then bootstrapped to ensure that they were robust to changes in composition. They fared fairly well, with an average re-assignment of less than 21%.

Hierarchical clustering can be very useful when comparing like a relatively small number of things. It allows the analyst to see hidden patterns and connections. The clusters just statistical artifacts; they describe real groupings of people who responded to the survey. These clusters will be carried through the entire analysis. Already of note are how the clusters are divided by gender (see Table I).

Figure 1. Figure II

CLUSTER	% Male	% Female
1	33	68
2	42	27
3	70	72
4	34	30
5	4	3
6	29	77
7	6	7
Total	170	294

Figure III

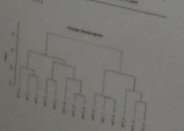


Figure IV

CLUSTER	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
6	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
7	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

### Analysis:

Look at Figures II. These are the density distributions for each of the 14 questions. The questions are in the right column under suitably descriptive headings.

Cluster 6 is the 'best' cluster—almost none of the answers in the cluster are in the 'wrong' direction.

Clusters 7 and 1 are the 'worst' clusters. Internal consistency is important in a survey; it measures how well different questions that intend to measure the same underlying dimension produce similar results. On average, similar questions should be answered similarly by each respondent. The correlation table (Table III) shows that the questions are correlated as we would expect if the survey were internally consistent. For each question, there are two rows, the top row is the correlation (red if positive) and the bottom is the p-value (bold if less than .10). The p-value here is the chance of getting a correlation that extreme by chance.

Figure II



Table III

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14
0.12	0.15	0.18	0.21	0.24	0.27	0.30	0.33	0.36	0.39	0.42	0.45	0.48	0.51
0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18

Look again to Appendix A; questions IATT\_6 - IATT\_10 are the best (most positively) correlated with 'agree' or 'strongly agree'. The opposite is true for the remaining questions. Using a hierarchical clustering method described in the text, variables grouped in the same cluster are expected (see Figure III).

Figure IV is a spatial coordinate plot. Many different dimensions/variables are placed along the horizontal axis. Each vertical axis is its own scale. In this case, only the index and cluster assignment had different axes than the 1-5. The best part about these plots is that you can trace any line across and see how one respondent answered all the questions.

A couple questions stood out as having interesting distributions when the cluster information is added in: IATT\_6 - IATT\_8 and IATT\_9. The first three are interesting because they all have nearly identical distributions; within these queries very similarly for each question. Of course, these questions deal with sexual assault. A depressingly large number of people think that it is not unacceptable behavior (see Re-II). Luckily, cluster 5 is also the smallest cluster. IATT\_9 asks whether sexist comments make the respondent uncomfortable. As seen in Figure 7, the responses for the clusters are all over the place. Cluster 6 responded far better than everyone else on this question, followed by clusters 1 and 3. These clusters have nearly the same distribution for this question. IATT\_9 is the most spread out of all of the first fourteen questions, although IATT\_10 is close second. Also of note is how many people think that it is ok to 'facebook stalk' their peers through their friends and acquaintances.

In Table IV, we can see the composition of clusters by different demographics. The 'problem' clusters have higher than average membership to Greek organizations. Those same clusters 6 and 7 also have much lower attendance in a

## Determining a Best Fit Distribution Function Across Foreign Market Sectors

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### Abstract

This paper describes the process of developing the best fit distribution function of foreign stocks. This work is an extension of the work done by Richard Kuper, student of Edward Thorp. The best fit distribution function is developed by comparing the Student's t-Distribution and Kuper's Student's t-Distribution. Actual values and predicted values obtained by utilizing the two different distributions are compared using the mean absolute percentage error (MAPE). The best fit distribution is defined by the function that results in the smallest MAPE. The process concludes that Kuper's Student's t-Distribution with 2.1 degrees of freedom was the best fit across the different market sectors.

### Method

1. Center the data about zero by considering the natural log of the wealth relative.

$$P_t = P_{t-1} + rP_{t-1}$$

$$P_t = (1+r)P_{t-1}$$

$$P_t/P_{t-1} = 1+r$$

$$\ln(P_t/P_{t-1}) = \ln(1+r)$$

2. Use a moving average to determine the mean and standard deviation for a given logged wealth relative.

3. Create a histogram of the logged wealth relatives with a predetermined interval radius.

4. Use the mean, standard deviation, and histogram heights to make predictions of the logged wealth relatives.

5. Run through different day counts and degrees of freedom for each t-distribution.

6. Compare the actual and predicted values using mean absolute percentage error (MAPE).

$$MAPE = \frac{\sum_{i=1}^n |d_i - A_i|}{n} \times 100\%$$

Student's T-Distribution Function:

$$f_t(x) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi v}} \left(1 + \frac{x^2}{v}\right)^{-(v+1)/2}$$

Kuper's Student's T-Distribution Function:

$$g_v(x) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}} \left(1 + \frac{x^2}{v-2}\right)^{-(v+1)/2}$$

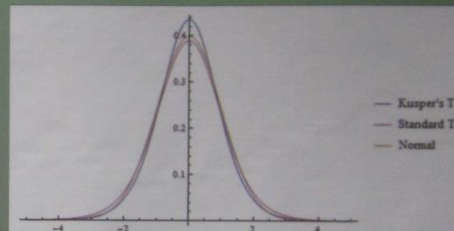


Figure 1: The Student's t-distributions approach the Normal distribution as degrees of freedom increase. Degrees of freedom = 10.

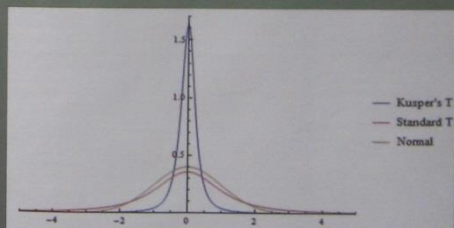


Figure 2: Degrees of freedom = 2.1

### Results

Using data from June 2002 to November 2013, the best fit distribution function was calculated for the following foreign stock market indices: Nikkei, FTSE, and DAX. For all three, Kuper's transformed Student's t-Distribution function with 2.1 degrees of freedom is the best fit. However the best day counts to calculate the moving averages are as follows:

Nikkei: 38 days  
FTSE: 36 days  
DAX: 40 days

### Conclusions and Further Research

Kuper's transformation of the Student's t-Distribution provides a better fit for the stock data. However, his use of 4.9 degrees of freedom was not validated in this study as 2.1 resulted in a better fit. Because Kuper used a 64 day moving average, which was not a result of the study, and because the day count is not consistent in the given results, further study includes testing the sensitivity to the number of days.

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# Characterizing Chaotic Functions: Devaney Chaos is Stronger Than Li-Yorke

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## ABSTRACT

In the field of modern chaos theory, it is a developmental stage. There is still no universal accepted definition for a chaotic function. Many different sets of criteria have been considered over the years. It is useful to examine the definitions we do have and how they relate to one another in the hopes of one day arriving at a universal definition for a chaotic function. We will examine Devaney's definition and characterize the logistic map in Devaney's chaotic. We will then compare this definition to the well-known Li-Yorke definition for chaos and see which set of criteria is stronger.

## Elements of Devaney Chaos

Let  $V$  be a set. Consider  $f: V \rightarrow V$

- $f$  has sensitive dependence on initial conditions
- Periodic points are dense in  $V$
- $f$  is topologically transitive

So  $f$  must be

- Unpredictable
- Regular
- Indecomposable

## Definitions

$f$  has sensitive dependence on initial conditions:  
 $\exists \delta > 0$  such that for any  $x \in V$  and any neighborhood  $N$   
of  $x$ , there exists  $y \in N$  and  $n \geq 0$  such that  
 $|f^n(x) - f^n(y)| > \delta$

Point  $x$  has period  $n$   
 $n$  is the smallest natural number such that  $f^n(x) = x$

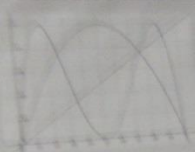
$f$  is topologically transitive:  
For any pair of open sets  $U, V \subset V$  there exists  $k \geq 0$   
such that  $f^k(U) \cap V \neq \emptyset$

## Example: The Logistic Map

$f(x) = rx(1-x)$ ,  $0 \leq x \leq 1$

We will consider the case where  $r = 4$  and determine whether this function is Devaney chaotic.

## Periodic Points are Dense in $[0,1]$



Periodic here on the curve  
 $f(x) = x$   
 $f^2(x) = x$   
 $f^3(x) = x$   
 $f^4(x) = x$   
 $f^5(x) = x$   
 $f^6(x) = x$   
 $f^7(x) = x$   
 $f^8(x) = x$   
 $f^9(x) = x$   
 $f^{10}(x) = x$   
 $f^{11}(x) = x$   
 $f^{12}(x) = x$   
 $f^{13}(x) = x$   
 $f^{14}(x) = x$   
 $f^{15}(x) = x$   
 $f^{16}(x) = x$   
 $f^{17}(x) = x$   
 $f^{18}(x) = x$   
 $f^{19}(x) = x$   
 $f^{20}(x) = x$

- Here we plot  $x, f(x), f^2(x), f^3(x), f^4(x), f^5(x), f^6(x), f^7(x), f^8(x), f^9(x), f^{10}(x), f^{11}(x), f^{12}(x), f^{13}(x), f^{14}(x), f^{15}(x), f^{16}(x), f^{17}(x), f^{18}(x), f^{19}(x), f^{20}(x)$
- Points of intersection represent points of period  $k \leq 20$
- Suggests  $f$  has dense set of periodic points in its open subset of  $[0,1]$



## Sensitive Dependence on Initial Conditions



The graph illustrates how sensitive the logistic map is to initial conditions. Two trajectories starting very close together diverge significantly over time, illustrating the butterfly effect.

## Method for Topological Transitivity

Proposition: A dense orbit implies topological transitivity.

Proof: Let  $f: V \rightarrow V$  be a function with a dense orbit. Let  $U, V \subset V$  be any pair of open sets. Let  $x \in U$  and  $y \in V$ . Let  $\{f^n(x)\}$  be a dense orbit. Then there exists  $k \geq 0$  such that  $f^k(x) \in V$ . Thus  $f^k(U) \cap V \neq \emptyset$ .

## Finding a Dense Orbit



The orbit of  $x$  is dense in  $V$  if and only if for every open set  $U \subset V$ , there exists  $n \geq 0$  such that  $f^n(x) \in U$ . This is equivalent to saying that the orbit of  $x$  is dense in  $V$  if and only if for every open set  $U \subset V$ , there exists  $n \geq 0$  such that  $f^n(x) \in U$ .

## Simplifying the Definition

The set  $V$  is a metric space. Topological transitivity can be defined in terms of open sets. The continuous function  $f: V \rightarrow V$  is topologically transitive if and only if for every open set  $U \subset V$ , there exists  $n \geq 0$  such that  $f^n(U) \cap U \neq \emptyset$ .

## Devaney vs. Li-Yorke

Necessary conditions for Li-Yorke chaos:  
 $f^2(x) = x$   
 $f^2(y) = y$   
 $|f^n(x) - f^n(y)| > \delta$   
 $|f^k(x) - f^k(y)| < \delta$   
 $|f^m(x) - f^m(y)| < \delta$



# Matrix Representation of $\mathbb{Z}[\sqrt{3}]$

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## Introduction

numbers. It is a ring that is defined as  $\mathbb{Z}[\sqrt{3}]$ .  
 Why is the presentation in the introduction  
 the reason for using matrices? As you  
 know, an element of the form  $a+bi$  is  
 lattice multiplication, on the other hand, is  
 efficient. Matrices are computer friendly,  
 is completed in a matter of seconds.

## Examples

$$a + b\sqrt{3} \mid a, b \in \mathbb{Z}$$

$$a^2 - 3b^2$$

is a multiplicative inverse in the

$$ab = ba = 1$$

$$a\gamma = \beta$$

## Theorems

$N(a) = N(b)$   
 unit if and only if  $N(a) = 1$   
 $\beta$  then  $N(a) \mid N(\beta)$

## Representation by Matrices

$$\alpha + \beta\sqrt{3} = x + y\sqrt{3}$$

$$= (x + 3y) + (3x + y)\sqrt{3}$$

$$\text{and } M_\alpha = \begin{bmatrix} x & 3y \\ 3x & y \end{bmatrix}$$

$$\begin{bmatrix} x + 3y & 3x + y \\ 3x + 3y & 3x + y \end{bmatrix} = M_{\alpha\beta}$$

Multiplication is preserved

$$M_\alpha + M_\beta = M_{\alpha+\beta}$$

## 1-1 Function

The function that maps  $a$  to  $M_a$  is also 1-1. That is, for every element  $M_a$ , there is exactly one element  $a$  that corresponds to it. We can show this fact by contradiction. Let's assume that for an  $M_a = \begin{bmatrix} a & 3b \\ 3c & a \end{bmatrix}$  there are two elements that correspond to it. Let these two elements be  $\alpha = a + r\sqrt{3}$  and  $\beta = a + s\sqrt{3}$ . For  $\alpha$  to correspond to  $M_a$ ,  $3r = 3b$  and  $3c = 3a$ . Similarly for  $\beta$  to correspond to  $M_a$ ,  $3s = 3b$  and  $3c = 3a$ . Thus, we can see that  $3r = 3s$  and  $3c = 3a$ . Therefore  $r$  and  $s$  are equal to each other. We can conclude that  $M_a$  corresponds to only one unique element  $a$  in  $\mathbb{Z}[\sqrt{3}]$ . This means that the function is 1-1.

## Faithful Representation

A faithful representation is a linear mapping where the function is a homomorphism and 1-1. As we have shown previously, the function that maps  $a$  to  $M_a$  satisfies both of these properties and is therefore a faithful representation.

$$a \in \mathbb{Z}[\sqrt{3}] \rightarrow \begin{bmatrix} a & 3b \\ 3c & a \end{bmatrix} \mid a, b, c \in \mathbb{Z}$$

## Using Matrices

The determinant of a 2x2 matrix  $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

$\det M_a = \det \begin{bmatrix} a & 3b \\ 3c & a \end{bmatrix} = a^2 - 3b^2 = N(a)$   
 We can clearly see that the determinant of  $M_a$  is equal to the Norm of  $a$ .  
 Therefore  $a$  is a unit if the determinant of  $M_a$  is  $\pm 1$  ( $N(a) = \pm 1$ )

The inverse of a unit,  $a$ , is also a unit by definition. What is the inverse of  $M_a$ ?

The inverse is equal to  $\frac{1}{\det M_a} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$  so the inverse of  $M_a$  is  $M_a^{-1}$

$$= \frac{1}{a^2 - 3b^2} \begin{bmatrix} a & -3b \\ -3c & a \end{bmatrix}$$

Example: Let  $a = 1 + 3\sqrt{3}$ . Then  $M_a = \begin{bmatrix} 1 & 9 \\ 9 & 1 \end{bmatrix} = M_a^{-1} = \frac{1}{1-27} \begin{bmatrix} 1 & -9 \\ -9 & 1 \end{bmatrix}$

$$M_a \cdot M_a^{-1} = \begin{bmatrix} 1 & 9 \\ 9 & 1 \end{bmatrix} \cdot \frac{1}{-26} \begin{bmatrix} 1 & -9 \\ -9 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

## Graph of units

Here are the units of  $\mathbb{Z}[\sqrt{3}]$  shown in red



## Norms of Multiplying with $a = 1 + \sqrt{3}$

Multiplying units on the graph by  $1 + \sqrt{3}$  will shift them as shown



Take 8 points and multiply them by  $1 + \sqrt{3}$ . The points follow the line of the hyperbola.



# QUADRATIC INTEGER RINGS AS SUBSETS OF THE REALS

Kaitlyn Rehberger  
Advisor: Andrew Clifford

## ABSTRACT

Project studies quadratic integer rings  $\mathbb{Z}[\sqrt{p}]$  ( $a, b \in \mathbb{Z}$ ), by viewing in order to get a more visual doing this, we can prove that one, and then use this method of multiples of the units of this ring,  $a, k \in \mathbb{Z}$  are dense in the real

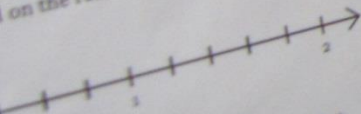
## $\mathbb{Z}[\sqrt{p}]$ IS DENSE IN $\mathbb{R}$

quadratic integer rings as a subset of the reals analytic way of looking at it in  $\mathbb{Z}[\sqrt{p}]$  is dense in  $\mathbb{R}$ .

elements in  $\mathbb{Z}[r]$ , where  $r$  is an irrational number that are close to the origin for any  $\epsilon > 0$  directly from the following lemma.

Let  $r$  be an arbitrary irrational number. Then for any  $\epsilon > 0$ , there exist pairs of  $m, n$  where  $m, n \in \mathbb{Z}$  such that  $|nr - m| < \epsilon$ .

Consider all multiples of the arbitrarily small  $\epsilon$  from the previous step, we can see that they are dense on the real line with a distance of  $|a| < \epsilon$ .



no matter what real number  $B$  or what  $\epsilon > 0$  we choose, there will always be an element in the  $\epsilon$ -neighborhood of  $B$ .

we that  $\mathbb{Z}[\sqrt{p}]$  is dense on the real line.

neighborhood of the real numbers will always contain an element  $\alpha = a + b\sqrt{p}$  within it. This gives us a neighborhood of  $B$  where these elements reside on the

## UNITS OF $\mathbb{Z}[\sqrt{p}]$

The set of all units of  $\mathbb{Z}[\sqrt{p}]$  form a subset of  $\mathbb{Z}[\sqrt{p}]$  and we can denote it by  $U_p$ .

To find units of a given quadratic integer ring, we need to find the integer solutions to Pell's equation,  $x^2 - py^2 = \pm 1$ .

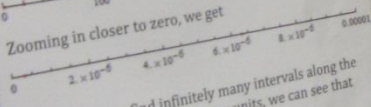
**Pell's Theorem:** If  $(x_1, y_1)$  is the smallest possible positive integer-valued solution to  $x^2 - py^2 = \pm 1$ , then every other solution  $(x_k, y_k)$  can be computed from the smallest solution by the formula  $(x_k, y_k) = (x_1 + y_1\sqrt{p})^k$ .

## $U_p$ IS NOT DENSE IN $\mathbb{R}$

$\alpha = 10 + 3\sqrt{11}$  is the unit in  $\mathbb{Z}[\sqrt{11}]$  with the smallest possible positive integer values for  $x$  and  $y$ .

Thus  $(10 + 3\sqrt{11})^k, k \in \mathbb{Z}$ , generates the rest of the units and we can plot them on the real line.

Zooming in closer to zero, we get



Because we can find infinitely many intervals along the real line that do not contain any units, we can see that  $U_p$  is not dense in  $\mathbb{R}$ .

## INTEGRAL MULTIPLES OF $U_p$ DENSE IN $\mathbb{R}$

$V_p = \{na, \alpha \text{ is a unit, and } n, k \in \mathbb{Z}\}$

**Lemma:** If  $S$  is a subset of the reals satisfying:

- $S$  is closed under integral multiplication
- $S$  has a zero limit point

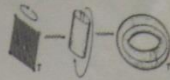
then  $S$  is dense.

**Overview of proof  $V_p$  is dense in  $\mathbb{R}$ :**

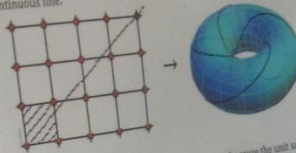
- 1) We need to show  $\exists$  elements in  $V_p$  near 0 for any  $\epsilon > 0$ .
- 2) Next, we consider all multiples of the set of units which are all evenly spaced on the real line with a distance of  $|a| < \epsilon$  apart. If we take the integer multiples of these elements, we can see how they will gradually fill the real line, with the ones landing between 0 and 1 having more impact.

## RELATION TO THE TORUS

Geometrically, we can obtain the torus  $T^2$  by gluing opposite edges of the integer lattice.



Say we have a line  $y=rx$  starting from  $(0,0)$ , where  $r$  is a rational slope. In this case we have  $r = \frac{1}{4}$ . When this lattice is mapped onto the torus, it is just one continuous line.



When  $r$  is irrational, this image will never start over because the unit square would be filled in almost completely with lines that never touch each other.

We can cut on the torus with an irrational slope of the form  $y = \sqrt{x}$ . Because the slope is irrational, the line will pass arbitrarily close to every point on the unit square, which in turn means it will pass arbitrarily close to every point on the torus.

This gives us another way of visualizing how the quadratic integer rings are dense.

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## A FUNCTION ON THE MODULI SPACE

by Kaitlyn

### ABSTRACT

### THE PROBLEM

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# Nonlinear Shooting Method to Solve Two-Point Boundary Value Problems that Arise in Reaction Engineering

Aakash Sheth  
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Department of Mathematics and Statistics, The College of New Jersey

## Introduction

Differential equations relate a function of one or more variables to its derivatives. Seen throughout a wide array of scientific disciplines, mathematics, and engineering applications, differential equations serve as an invaluable tool for modeling and helping solve physical problems. Generally, the simplest type of differential equations are initial value problems. However, in order to model more complex and complicated scenarios, it is necessary to solve boundary value problems. Two point boundary value problems (TPBVPs) are those that involve Dirichlet boundary conditions and those that involve Neumann boundary conditions. Dirichlet problems consist of a BVP and a fixed boundary condition. For example, the Dirichlet boundary conditions for a reaction are  $y(a) = \alpha$  and  $y(b) = \beta$ . For a Neumann problem if a relation exists it is unique. Neumann boundary conditions specify values that result in derivative boundary conditions. For a BVP, Neumann boundary conditions on the interval  $[a, b]$  take the form  $y'(a) = \alpha$  and  $y'(b) = \beta$ . If a solution exists for a Neumann problem, it is unique to within an arbitrary constant. The shooting method is a technique in numerical analysis for solving both types of TPBVPs. It involves using the boundary conditions to reduce the TPBVP to a much more approachable initial value problem. It is especially applicable to solving TPBVPs that arise in reacting flow.

## Main Objectives

1. Demonstrate NSM for a TPBVP with an exact solution
2. Use NSM to construct solutions to the premixed flame regime structure for ideal gases

## Method of Solution

MATLAB2013B was utilized to complete all graphs and analysis. Matlab's subroutine ode45 was used to exactly solve and find the graphical solution for Example 1. In order to numerically solve both BVPs via NSM, a separate Runge-Kutta subroutine was written and called. For the nonlinear boundary value problem of the form:

$$y'' = f(x, y, y'), a \leq x \leq b$$

$$y(a) = \alpha$$

$$y(b) = \beta$$

In Example 2, the following conversions were utilized to generate two initial value problems:

$$y'' = f(x, y, y'), a \leq x \leq b$$

$$y(a) = \alpha$$

$$y'(a) = \gamma$$

$$y'' = f(x, y, y'), a \leq x \leq b$$

$$y(a, \gamma) = 0$$

$$y'(a, \gamma) = 1$$

for  $y(a)$  and  $y'(a)$ . Update

$$\gamma_{new} = \gamma - \frac{y(b, \gamma) - \beta}{s(b, \gamma)}$$

## Results

Example 1

The BVP below was approached numerically using the NSM, with the number of iterations imposed on the exact analytical solution.

Example BVP

$$y'' + \frac{2yy'}{(y+x^2)^2} = 0, p = 10^{-5}$$

Boundary Conditions

$$y(-0.1) = \frac{-0.1}{\sqrt{p+0.1}}$$

$$y(+0.1) = \frac{0.1}{p+0.1}$$

Exact Solution

$$y(x) = \frac{x}{\sqrt{p+x^2}}$$

Converting the BVP problem to an IVP problem, the two boundary conditions are used to generate a system of four equations.

$$y'_1 = y_2$$

$$y'_2 = \frac{-2yy_2}{(p+x^2)^2}$$

$$y'_3 = y_4$$

$$y'_4 = \frac{-2yy_4}{(p+x^2)^2}$$

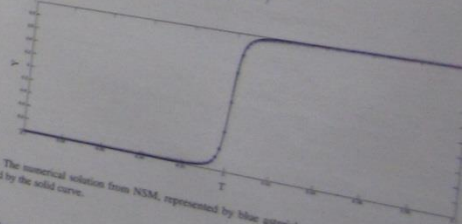


Figure 1: The numerical solution from NSM, represented by blue asterisks, superimposed on the exact solution represented by the solid curve.

Example 2

In the following example, we analyzed the behavior of a premixed flame or a flame where the oxidizer has been mixed with fuel before reaching the flame front. The specific premixed case we looked at was frozen flow, conditions that allow for the minimum thermodynamic case to be converted to kinetic energy, on one side of the flame and equilibrium flow, the chemical reactions are fast enough to always maintain thermodynamic equilibrium, on the other side of the flame.

Example BVP

Boundary Conditions

$$y'' = \frac{2y''(y+mx)}{2}$$

$$\lim_{x \rightarrow -\infty} y' = -1$$



# Relations Among Moduli Spaces of Triangles

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### Introduction

Moduli spaces are geometric representations of sets of mathematical objects. Here I consider a collection of moduli spaces of Euclidean triangles and consider their relationships to one another and to the triangles they intend to represent. In so doing, I arrive at a better understanding of the structure of the various moduli spaces of triangles and their value in representing Euclidean triangles.

### Moduli Spaces

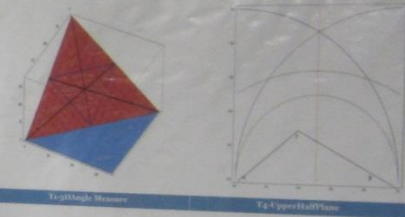
A moduli space requires:  
Objects to represent  
Parameters to sort these objects  
A notion of equivalence among the objects



### Moduli Spaces of Triangles

Angle Measure	T2-2D Angle Measure
$\alpha, \beta, \gamma \in \mathbb{R}^+ \text{ and } \alpha + \beta + \gamma = 180$	$\alpha, \beta \in \mathbb{R}^+ \text{ and } \beta < 180 - \alpha$
Side Measure	T4-UpperHalfPlane
$a, b, c \in \mathbb{R}^+ \text{ and } a + b > c, a + c > b, b + c > a$	$\alpha, \beta \in (0, \pi) \text{ and } \alpha + \beta < \pi$

### Are these spaces equivalent?



To ask whether two moduli spaces are equivalent is to ask whether or not these spaces represent their objects with the same fidelity.  
 ❖ Are these apparently different representations capturing the set of Euclidean triangles in a comparable and unbiased way?

### Investigations:

- ❖ Given a triangle, what are the chances that it is obtuse? - Calculation 1
- ❖ What is the average product of interior angles for Euclidean triangles - Calculation 2

Space	Calculation 1	Product
T3	$\frac{\int_{1/4}^{3/4} \int_{1/2-a}^a db da + \int_{1/2}^{1/2} \int_{1/2-a}^{2a-1} db da}{\int_{1/4}^{3/4} \int_{1/2-a}^a db da + \int_{1/2}^{1/2} \int_{1/2-a}^{2a-1} db da}$	$\approx 0.6822 = \frac{3}{4}$
T4	$\frac{\int_0^{1/2} \int_0^{1/2-a} dy dx}{\int_0^{1/2} \int_0^{1/2-a} dy dx}$	$\approx 0.6391 = \frac{3}{8}$
Space	Calculation 2	Product
T3	$\frac{\int_{1/4}^{3/4} \int_{1/2-a}^a \alpha(\alpha, \beta) \beta(\alpha, \beta) da db + \int_{1/2}^{1/2} \int_{1/2-a}^{2a-1} \alpha(\alpha, \beta) \beta(\alpha, \beta) da db}{\int_{1/4}^{3/4} \int_{1/2-a}^a da db + \int_{1/2}^{1/2} \int_{1/2-a}^{2a-1} da db}$	$\approx 119923 \approx 97200$
T4	$\frac{\int_0^{1/2} \int_0^{1/2-a} \alpha(x, y) \beta(x, y) r(x, y) dx dy}{\int_0^{1/2} \int_0^{1/2-a} dx dy}$	$\approx 119612 \approx 97200$

\* The functions  $\alpha, \beta, \gamma$  take the parameters of one space (T3 or T4) and produce the parameters of the other space (T4 or T3).  
 T3 represented by the triangles indicated in T3 or T4.

- ❖ Calculations on T3 and T4 are inconsistent with T2.
- ❖ The reason for this is that spaces T3 and T4 are not equivalent.
- ❖ To correct this problem the coordinates must be rectified to balance the coordinate systems.

T3-Space  
 $J_3 = \frac{1}{ab}$

T4-UpperHalfPlane  
 $J_4 = \pi^2(x^2 + y^2)$

Space	Calculation 1
T3	$\frac{\int_{1/4}^{3/4} \int_{1/2-a}^a J_3 db da + \int_{1/2}^{1/2} \int_{1/2-a}^{2a-1} J_3 db da}{\int_{1/4}^{3/4} \int_{1/2-a}^a J_3 db da + \int_{1/2}^{1/2} \int_{1/2-a}^{2a-1} J_3 db da}$
T4	$\frac{\int_0^{1/2} \int_0^{1/2-a} J_4 dy dx}{\int_0^{1/2} \int_0^{1/2-a} J_4 dy dx}$

\* Where the ellipsis notes the formula from the Jacobian correction

### Discussion/Conclusion

It is observed that not all moduli spaces are equivalent. Despite this, it has been shown that dimension may be transferred with the appropriate calculations.  
 ❖ It remains to be shown whether these moduli spaces are geometric representations of Euclidean triangles. Quantities manifest in the structure of these spaces are...

# A Comparison of Subrings: $\mathbb{Z}[\sqrt{17}]$ and $\mathbb{Z}[\sqrt{19}]$

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Advisor: Andrew Clifford

## Abstract

This project analyzes divisibility in  $\mathbb{Z}[\sqrt{17}]$  and  $\mathbb{Z}[\sqrt{19}]$ . Specifically, we consider when prime integers can be factored and which are irreducible. By analyzing the  $p$ -norm integers modulo 4, we see how the factorization behaves differently in each subring. Also, we use norms to guarantee irreducibility for the primes that could not be factored.

## Definitions and Properties

- $\mathbb{Z}[\sqrt{p}] = \{a + b\sqrt{p} \mid a, b \in \mathbb{Z}\}$ 
  - $p = 17$  and  $p = 19$
- Norm
  - $N(a + b\sqrt{p}) = a^2 - pb^2$
  - $N(\alpha\beta) = N(\alpha)N(\beta)$
- Units: If  $\alpha$  is a unit
  - $\alpha + \alpha^{-1} = 1$
  - $N(\alpha) = \pm 1$
- Divisibility
  - If  $\alpha \mid \beta$  then  $N(\alpha) \mid N(\beta)$
- Quadratic Reciprocity
  - A nonzero number congruent to a square mod  $p$  is called a quadratic residue mod  $p$
  - Legendre Symbol
    - $\left(\frac{a}{p}\right) = 1$  iff  $a$  is a quadratic residue mod  $p$
    - $\left(\frac{a}{p}\right) = -1$  iff  $a$  is not a quadratic residue mod  $p$
  - Law of Quadratic Reciprocity
    - $\left(\frac{p}{q}\right) = \begin{cases} \left(\frac{q}{p}\right) & \text{if } q \text{ or } p \equiv 1 \pmod{4} \\ -\left(\frac{q}{p}\right) & \text{if } q \text{ and } p \equiv 3 \pmod{4} \end{cases}$

## Norms Modulo 4

What norms are possible for each subring?  
 $a^2 - pb^2 \equiv n$

$n$	$p = 17$	$p = 19$
$a^2 - 17b^2 \equiv n \pmod{4}$	$n \equiv 0, 1, 3 \pmod{4}$	$n \equiv 0, 1, 3 \pmod{4}$
$a^2 - 19b^2 \equiv n \pmod{4}$	$n \equiv 0, 1, 2 \pmod{4}$	$n \equiv 0, 1, 2 \pmod{4}$

## Factoring Prime Integers

Which primes can be factored in  $\mathbb{Z}[\sqrt{17}]$  or  $\mathbb{Z}[\sqrt{19}]$ ?  
Example:  $13 = (9 - 2\sqrt{17})(9 + 2\sqrt{17})$   
Note:  $N(9 - 2\sqrt{17}) = N(9 + 2\sqrt{17}) = 13$   
Will this always be the case?  
If  $a \mid q$  then  $N(a) \mid N(q)$   
 $N(q) = q^2$   
 $\Rightarrow N(a) = \pm q$

## Table of Factorizations

Primes	$\mathbb{Z}[\sqrt{17}]$		$\mathbb{Z}[\sqrt{19}]$	
	Positive Norms	Negative Norms	Positive Norms	Negative Norms
2	?	?	?	?
3	?	?	?	$(13 + 3\sqrt{19})(-13 + 3\sqrt{19})$
5	?	?	$(9 + 2\sqrt{19})(9 - 2\sqrt{19})$	$(4 + \sqrt{19})(-4 + \sqrt{19})$
7	?	?	?	?
11	?	?	?	?
13	$(9 - 2\sqrt{17})(9 + 2\sqrt{17})$	$(2 + \sqrt{17})(-2 + \sqrt{17})$	?	?
17	$(17 + 4\sqrt{17})(17 - 4\sqrt{17})$	$(\sqrt{17})(\sqrt{17})$	?	?
19	$(6 + \sqrt{17})(6 - \sqrt{17})$	$(7 + 2\sqrt{17})(-7 + 2\sqrt{17})$	$(6 + \sqrt{19})(6 - \sqrt{19})$	?
23	?	?	?	$(\sqrt{19})(\sqrt{19})$
29	?	?	?	?
31	?	?	?	?
37	?	?	?	?
41	?	?	?	$(10 + 7\sqrt{19})(-30 + 7\sqrt{19})$
43	?	?	?	$(13 + 3\sqrt{19})(-13 + 3\sqrt{19})$
47	$(14 + 3\sqrt{17})(14 - 3\sqrt{17})$	$(5 + 2\sqrt{17})(-5 + 2\sqrt{17})$	$(9 + 2\sqrt{19})(9 - 2\sqrt{19})$	$(4 + \sqrt{19})(-4 + \sqrt{19})$

## Alternating Factorizations

In  $\mathbb{Z}[\sqrt{19}]$  norms are  $\equiv 0, 1, 2 \pmod{4}$   
If  $q \equiv 1 \pmod{4}$   
Want factors with norms  $\pm q \equiv \pm 1 \pmod{4}$   
 $1 \equiv 1 \pmod{4}$  ✓  
 $-1 \equiv 3 \pmod{4}$  ✗  
If  $q \equiv 3 \pmod{4}$   
Want factors with norms  $\pm q \equiv \pm 3 \pmod{4}$   
 $3 \equiv 3 \pmod{4}$  ✗  
 $-3 \equiv 1 \pmod{4}$  ✓  
In  $\mathbb{Z}[\sqrt{17}]$  norms are  $\equiv 0, 1, 3 \pmod{4}$   
Norms of  $\pm q$  are possible

## Is 3 irreducible

Can 3 be a norm?  
 $a^2 = 3 + 17b^2$   
Modulo 3  
 $a^2 \pmod{3} \equiv 0 + 2b^2 \pmod{3}$   
Squares in modulo 3 are either 0 or 1  
Legendre Symbol  
 $\left(\frac{17}{3}\right) = \left(\frac{2}{3}\right) = -1$   
3 is irreducible in  $\mathbb{Z}[\sqrt{17}]$

## Table of Irreducibles

Primes	$\mathbb{Z}[\sqrt{17}]$	
	Positive Norms	Negative Norms
2	IRREDUCIBLE	IRREDUCIBLE
3	IRREDUCIBLE	IRREDUCIBLE
5	IRREDUCIBLE	IRREDUCIBLE
7	IRREDUCIBLE	IRREDUCIBLE
11	IRREDUCIBLE	IRREDUCIBLE
13	$(9 - 2\sqrt{17})(9 + 2\sqrt{17})$	$(2 + \sqrt{17})(-2 + \sqrt{17})$
17	$(17 + 4\sqrt{17})(17 - 4\sqrt{17})$	$(\sqrt{17})(\sqrt{17})$
19	$(6 + \sqrt{17})(6 - \sqrt{17})$	$(7 + 2\sqrt{17})(-7 + 2\sqrt{17})$
23	IRREDUCIBLE	IRREDUCIBLE
29	IRREDUCIBLE	IRREDUCIBLE
31	IRREDUCIBLE	IRREDUCIBLE
37	IRREDUCIBLE	IRREDUCIBLE
41	IRREDUCIBLE	IRREDUCIBLE
43	$(14 + 3\sqrt{17})(14 - 3\sqrt{17})$	$(5 + 2\sqrt{17})(-5 + 2\sqrt{17})$
47	$(8 + \sqrt{17})(8 - \sqrt{17})$	$(15 + 4\sqrt{17})(-15 + 4\sqrt{17})$

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# Modeling Baseball HOF Selection and Worthiness: A Case Study

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## Introduction

Major League Baseball (MLB) is the sports league from 1907-2004 and is widely considered to be the greatest ballgame in the history of the game. A player who fits in either of the below categories did not play competitively in the field. Some of the most notable accomplishments include being two-time American League (AL) batting champion, seven-time AL MVP and two-time AL Star (Silver Slugger Award winner). On January 11, 2024, Mike Trout became the first player to be elected to the Hall of Fame (HOF) as a 36-year-old. The Hall of Fame is a prestigious honor and is considered the highest honor in baseball. The Hall of Fame is a place where the best players are inducted into the Hall of Fame. The Hall of Fame is a place where the best players are inducted into the Hall of Fame. The Hall of Fame is a place where the best players are inducted into the Hall of Fame.



The goal of this research is to create a mathematical model for HOF-worthiness on an individual player level. The underlying question is how to do this. One player's worthiness to be inducted into the HOF is a complex question. It is not just about how many runs a player has scored or how many home runs he has hit. It is about how well a player has performed in all aspects of the game. It is about how well a player has performed in all aspects of the game. It is about how well a player has performed in all aspects of the game.

Using data on player stats (such as batting average, on-base percentage, slugging percentage, and other statistics) and the number of years a player has played in the major leagues, we will attempt to build a model that accurately predicts the language used by the BBWAA. All aspects of baseball will be included in the model. All statistics and data will be collected from <https://baseball-reference.com>. All aspects will be done through <https://baseball-reference.com>.

## Selecting Variables

**Player Name, Playing Ability**  
 Consider the 12 offensive statistics through a Principal Component Analysis (PCA) to determine which variables are most important in the model. The 12 offensive statistics are: Runs (R), Hits (H), Total Bases (TB), Runs Scored (RS), Batting Average (BA), On-Base Percentage (OBP), Slugging Percentage (SLG), Total Fielding Percentage (TFP), Total Error (TE), Total Outs (TO), Total Inning (TI), Total Outs (TO), Total Inning (TI), Total Outs (TO), Total Inning (TI).

## Methods: PCA, Binary Logistic Regression

**Principal Component Analysis (PCA)**  
 PCA is a multivariate dimension reduction method. It takes a set of highly correlated variables and reduces them to a few uncorrelated variables which are linear combinations of the original variables. The new variables are ordered by amount of variance accounted for in the original data. The coefficients assigned to the original variables in the principal component scores often have intuitive meanings. PCA helps to explain the underlying structure of the complicated data set.

**Binary Logistic Regression (BLR)**  
 BLR is employed when we want to use independent variables to predict a discrete, binary outcome. There are no assumptions about the distribution of the predictor variables. The probability of having the discrete outcome is where  $X = (X_1, X_2, \dots, X_n)$  is a linear combination of the independent variables.

## PCA Results

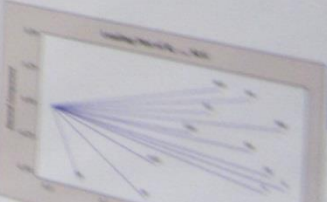
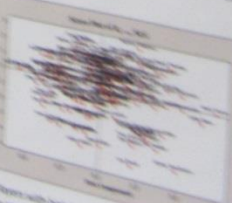
The first 10 Principal Component Scores account for 98.23% of variation in data. Variables (after 10) are: TB, RS, BA, OBP, SLG, TFP, TE, TO, TI.

PCA is a contrast between power hitters and small-ball players.  
 PCA is a contrast between OBP and SLG vs. TFP vs. TO, TI vs. BA, OBP, SLG.

We have successfully reduced 12 highly correlated offensive statistics into 2 uncorrelated statistics with intuitive meanings.

Principal Component Analysis: R, H, TB, RS, BA, OBP, SLG, TFP, TE, TO, TI

Component	R	H	TB	RS	BA	OBP	SLG	TFP	TE	TO	TI
1	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
2	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
3	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
4	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
7	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
8	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
9	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
10	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01



Before running a BLR, we performed a PCA to look for any possible correlations between the variables. The results show that the variables are highly correlated. This means that there is a lot of redundancy in the data. We now have 71 indices players in the data set. We will use a model and regression to predict the outcome.

**Model 1:**  
 - Includes variables: BBWAA, Electro, P-values for 5-10, indicate model fit, BBWAA, Concorde, Edgar, Wainwright, QUESADA, NASTI  
 - Although the model could be lower.

**Model 2:**  
 - After eliminating variables, we have variable significance level.  
 - P-values for 5-10, model fit, BBWAA, Concorde, Edgar, Wainwright, QUESADA, NASTI

## Discussion

According to our eliminating variables, some variables were

# What Does $SL_2(\mathbb{R})$ Look Like?

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Every Point on the Interior of the Solid Torus is a Matrix



Figure 5: Theta Specifies a Disk



Figure 6: phi Specifies a Circle



Figure 7: psi Specifies a Point on a Circle

How does  $K_\theta$  act on the interior of the solid torus?

Let  $K_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \in K$ .

$$K_\theta \cdot A_{\theta, \phi, \psi} = A_{\theta, \phi, \psi}$$

Interpretation

- $K_\theta$  rotates the torus by  $\theta$  radians counterclockwise.

How does  $A_\phi$  act on the interior of the solid torus?

Let  $A_\phi = \begin{pmatrix} 1 & \phi \\ 0 & 1 \end{pmatrix} \in A$ .

$$A_\phi \cdot A_{\theta, \phi, \psi} = A_{\theta, \phi, \psi}$$

Interpretation

- $A_\phi$  contracts/expands the blue circles interior to the

$$A_\nu \cdot A_{\theta, \phi, \psi} = A_{\theta, \phi, \psi}$$

Observations

- $\nu = \frac{1}{\sqrt{1-\phi^2}}$  only depends on  $\phi$
- $A_\nu$  preserves disk structure

How does  $N_\psi$  act on the interior of the solid torus?

Let  $N_\psi = \begin{pmatrix} 1 & \psi \\ 0 & 1 \end{pmatrix} \in N$ .

$$N_\psi \cdot A_{\theta, \phi, \psi} = A_{\theta, \phi, \psi}$$

Interpretation

- $N_\psi$  shifts points around a blue circle

$$N_\psi \cdot A_{\theta, \phi, \psi} = A_{\theta, \phi, \psi}$$

Observations

- $\psi = \frac{1}{\sqrt{1-\phi^2}}$  only depends on  $\phi$
- $N_\psi$  preserves disk structure

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# Using Mathematical Modeling To Study the Predator-Prey Relations Lions, Wildebeest, and Buffalo in the Ngorongoro Crater Area

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## Abstract

The model created aims to study the population dynamics of a three-species system between the lion, blue wildebeest, and buffalo in the Ngorongoro Crater area of Tanzania. The model mainly focuses on studying the interaction between these populations using the Lotka-Volterra system of equations. Growth/decline rates were estimated for each population using the Lotka-Volterra system, and solutions were then approximated using the Euler method in order to determine the predicted population size for each species. By analyzing error using the mean standard deviation, mean actual deviation, and mean actual percentage error, the parameters of the Lotka-Volterra system were estimated. This included the prey growth rate ( $r$ ), predation rate(s), the efficiency of conversion from prey to predator ( $c$ ), and the predator death rate in the absence of prey ( $d$ ).

A test was also used to study the impact of the wet season (Nov-Apr) vs. dry season (May-Oct) on the total wildebeest and buffalo population. Since these species are the lions main food source, by analyzing the change in these prey for each season we are also able to determine the significance of season on the lion population as well.

## Introduction

The Ngorongoro Crater is a 100 square mile landmark located in the Ngorongoro Conservation Area in Tanzania. It is one of the world's largest inactive, intact, and unfilled volcanic calderas, meaning that it formed from a volcano that collapsed after an eruption. The crater is home to a vast number of species including rhinoceros, wildebeest, buffalo, and zebra. It also has one of the densest lion populations known with a population of 123 recorded in 1998, though since then the population has decreased and usually around 50-100 lions. The lions and many ungulates living in the crater form a complex system of predator-prey relationships which affect the crater's ecosystem as a whole.

A Lotka-Volterra model describing this predator-prey relationship was used to study lion and ungulate behavior. Results of the model showed that the dynamics of the ungulate population is based largely on the dynamics of lions. In 1998 for instance, the lion population was 123 lions due to a combination of tick-borne disease and a wetter than average year, wildebeest and buffalo populations more than doubled, and then slowly declined to normal population levels. These findings are of great biological importance and can be used to better understand ungulate behavior.



located in the Ngorongoro Conservation Area in Tanzania. It is one of the world's largest inactive, intact, and unfilled volcanic calderas.

## Methods

The Lotka-Volterra system of equations was used to study the effect of the lion population on wildebeest and buffalo populations, and vice versa. The Lotka-Volterra system of equations are given by:

$$\frac{dN(t)}{dt} = N(t)(r - aP(t))$$
$$\frac{dP(t)}{dt} = P(t)(c\alpha N(t) - d)$$

A Mathematica model was created for this system, allowing population growth to be estimated using parameters input by the user as well as the predator and prey population at time  $t$ .

Solutions of the Lotka-Volterra system were approximated using the Euler method. This allowed the population size of each group to be estimated at time  $t+1$ . Parameters of the model were estimated by comparing the output of the Lotka-Volterra system to a series of actual population data for lions, blue wildebeest, and buffalo collected between 1989-2002. The MSD, MAD, and MAPE error statistics were calculated in order to assess the fit of the parameters.

$$MSD = \frac{\sum (y_i - \hat{y}_i)^2}{n}$$
$$MAD = \frac{\sum |y_i - \hat{y}_i|}{n}$$
$$MAPE = \frac{\sum \frac{|y_i - \hat{y}_i|}{|y_i|}}{n} \times 100\%$$

Statistical analysis was also used to better understand the data. A T-test was used to test the significance of the wet vs dry season on the wildebeest and buffalo populations. By allowing us to estimate the difference in prey populations, this indirectly allowed us to study the effect of season on the lion population. An ANOVA table was also created for further analysis.

Lion, Wildebeest, and Buffalo Population in Ngorongoro Crater Area



Figure 1: Lion, Blue Wildebeest, and Buffalo populations from 1989-2002. The average of wet and dry season populations were used for the wildebeest and buffalo populations. Lion populations tend to stay below 100, while the wildebeest and buffalo populations tend to stay in the thousands.

## Results

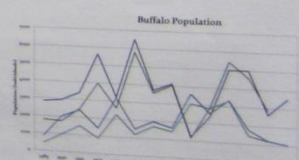
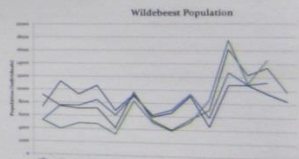
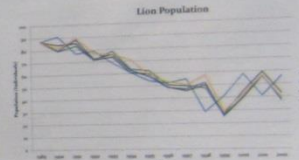


Figure depicts Lotka-Volterra dry season population dry season years. population

## Discussion

Results from the Lotka-Volterra model and parameter estimation gave significant insight into the Ngorongoro Crater ecosystem. Using minimum error as a basis for parameter estimation, the population size of the lion population was around 0.01 for wildebeest and 0.1 for buffalo. This is a fairly low mortality rate compared to the wildebeest population. The predation rate for buffalo was 10 times that of wildebeest, explaining the higher mortality rate. The death rate for lions was only around 0.1, which is one must also consider that the lion population is much smaller. The growth rate for both prey, the growth rate increased during wet season. Wildebeest and buffalo populations showed that season did not have a significant effect on the population have been known to increase and decreasing from wet to dry seasons, increased reproductive success during the wet season. However, the population did decrease every year during the wet season. How the population of the buffalo population...

# Comparative Study of Portfolio Risk Management Using Alternative Allocation Strategies

Alissa Migliore & Faculty Advisor: Dr. Edward Conjura

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## Abstract

This project is based on analyzing a portfolio with different types of investments... through changing allocations of investments in different equity classes... different types of assets that will react differently under the same trading and data with the question of how much risk an investor should take... the question of how much risk an investor should take... the question of how much risk an investor should take...

## Introduction

Developed by J. Welles Wilder, it is a very popular momentum indicator and changes in price movements of an investment. It is calculated on a 14-day time frame, but can vary to suit the needs of an investor is trying to analyze. Wilder's original formulation of RSI, uses a simple moving average of closing prices to calculate the average gain/loss for an investment. RSI is not data length dependent. RSI is calculated as follows:  $RSI = \frac{Average\ Gain}{Average\ Gain + Average\ Loss} \times 100$

Low parameters which indicate potential price swings... when the RSI rises above 70 and falls below 30... RSI is calculated as follows:  $RSI = \frac{Average\ Gain}{Average\ Gain + Average\ Loss} \times 100$

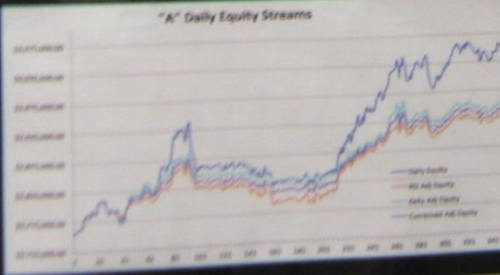
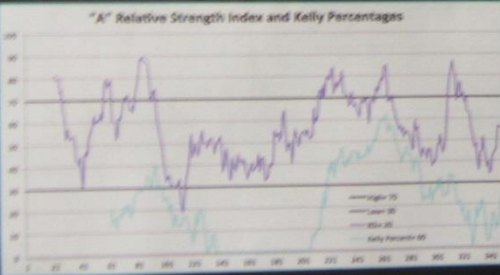
Developed by John Kelly, who worked for AT&T's Bell Labs... Kelly's formula is used to determine the optimal trade size... Kelly's formula is used to determine the optimal trade size...

It is equal to the percentage of trades that are winning... Kelly's formula is used to determine the optimal trade size... Kelly's formula is used to determine the optimal trade size...

## Methods

The data used in this project was provided by Dr. Conjura. The data consists of 4 different equity portfolios entitled "A", "S", "T", and "V". The data begins in 1991 and ends in 2012, and contains closed & daily equity values, in which we will be using the daily equity values. The VBA program receives the data input from the user, as well as user inputs for RSI length, RSI High & Low Parameters, and the Kelly Length. The program then creates a statistics sheet that includes the calculated RSI values, Kelly values, and then the adjusted equity values for 3 new equity streams. With the help of Dr. Conjura, we formulated different strategies for when to cut and add to the investor's position size. The first adjusted equity stream is created strictly using the RSI inputs. When RSI values cross the oversought level and return back down, the investor's position is cut to 50%, and returns back to 100% once the oversought level has been crossed below and then crossed above. In the Kelly adjusted equity stream, if the Kelly value < 0.5, then position size = 50%, if the Kelly value > 0.5, then position size = 100%. In the Combined adjusted equity stream, the weight used is the average of the RSI weight and Kelly weight, each respectively calculated in the other 2 equity streams.

Data Sheet	A*
RSI Length	30
RSI High Parameter	70
RSI Low Parameter	30
Kelly Length	60
<b>Calculate</b>	



## Results

To analyze the newly created equity streams, the data was run through a program written by Dr. Conjura. It returns an annual analysis, including a Profits/Decline ratio as well as the Sharpe Ratio. To an investor, the slightly more important result is the R/D ratio because it explains the largest decline that can be expected in relation to total profits. We want the ratios to increase in size. There were 5 different strategies analyzed which involved combining different RSI parameters together and separately changing the Kelly weight. Combined adjusted equity stream using both RSI & Kelly inputs turned out not to be an optimal trading strategy; it was formulated using the average weight of the RSI weights and therefore did not improve overall performance, but fell somewhere between the two methods. The results of the 5 different strategies used on the original 4 portfolios is shown below. The largest ratios are highlighted.

	Profits/Decline	Sharpe Ratio
Original A	5.5409	0.1724
RSI 20: 70/30	8.2139	0.2169
RSI 20: 55/35	6.7948	0.2135
RSI 14: 70/30	8.3807	0.2215
RSI 14: 55/35	6.9122	0.1808
Kelly 50	5.5736	0.1688
Kelly 40	6.1329	0.1504

	Profits/Decline	Sharpe Ratio
Original T	1.7225	0.0748
RSI 20: 70/30	1.6335	0.0788
RSI 20: 55/35	1.8175	0.0877
RSI 14: 70/30	1.3960	0.0624
RSI 14: 55/35	1.0364	0.0527
Kelly 50	1.6126	0.0698
Kelly 40	2.2324	0.0814

	Profits/Decline
Original S	3.817
RSI 20: 70/30	3.0161
RSI 20: 55/35	3.3708
RSI 14: 70/30	3.4835
RSI 14: 55/35	3.1134
Kelly 60	3.6764
Kelly 40	3.5648

	Profits/Decline
Original V	5.0709
RSI 20: 70/30	5.1004
RSI 20: 55/35	4.4443
RSI 14: 70/30	4.9098
RSI 14: 55/35	5.3892
Kelly 50	5.6674
Kelly 40	4.7731

## Conclusion

Portfolio "A" clearly performed best with a 14-day RSI under 70... strategies performed on Portfolio "S" resulted in smaller P/D values... increases in the Sharpe Ratio. I would recommend using the 20-day conditions on Portfolio "S". Portfolio "T" saw a significant increase... using the Kelly strategy at a 60-day length. Portfolio "V" saw the... cut, indicating that further analysis could be done to calculate... Another program could be written that would simulate many different... & Kelly values to achieve an optimal allocation technique. Also, the... used to actually formulate new equity streams could be adjusted...

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<http://www.investopedia.com/articles/technical/07/1801.asp>  
<http://www.investopedia.com/articles/technical/03/042203.asp>  
[http://stockcharts.com/help/faq.php?faq=chart\\_school/technical\\_indicators](http://stockcharts.com/help/faq.php?faq=chart_school/technical_indicators)  
<http://www.tradestation.com/education/technical-analysis-concepts-concepts>  
<http://www.investopedia.com/articles/technical/04/041004.asp>

# Applying Predator-Prey Techniques in an Attempt to Model a Relation Between New Zealand's Tui, and Australia's Brushtail Possum



Lisa Moen, Faculty Advisor: Dr. Ed Conjura  
 Mathematics Department, The College of New Jersey

## Abstract

This project completes a population dynamic study between two species, New Zealand's Tui bird and Australia's Brushtail Possum. The dynamics between these species is an example of indirect predation with the Kowhai tree located in the middle of the food chain. The Lotka-Volterra Predator-Prey model are sets of differential equations used to map the relationship between two species populations. Data evaluated using a Matlab R2014a program designed with the Lotka-Volterra model was used to complete a current and predatory time series analysis of this relationship.

## Introduction

New Zealand is a beautiful nation with an extremely fragile ecosystem. The Kowhai Tree is a national icon. This tree while widely distributed throughout the islands is near extinct in certain areas. This decline in population is likely to have been brought about by the introduction of Australia's Brushtail Possum. The opportunistic feeding habits of the Possum directly affect not only the plants on which they feed but also the native animals, like the Tui bird, that rely on those plants for food.

The Lotka-Volterra differential equations developed by Alfred Lotka and Vito Volterra measure the interactions between a predator and its prey over a period of time. These equations can be used to then predict the prolonged interaction between these species.

$$\frac{dx}{dt} = \alpha x_1 - \beta y_1 x_2$$

$$\frac{dy}{dt} = -\gamma y_2 + \delta y_1 x_2$$

Parameters alpha, beta, gamma, and delta > 0 determine the accuracy of the modeled data with the actual data.

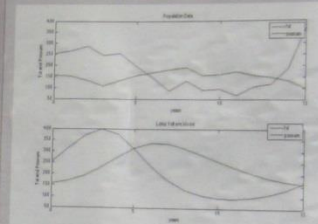
## Methods

- Data for Brushtail Possum and Tui populations collected from:
  - Landcare Research
  - ICUN RedList
  - The Department of Conservation in New Zealand
- Four unknown parameters posed a problem for mapping the relationship between the Tui and the Possum.
- A program created using MatLab R2014a established a relationship between species using the Lotka-Volterra differential equations.
  - Compared Tui and Possum populations directly.
  - Calculate error to find the appropriate parameter values.
  - Plot population relation.
- Compare data to intermediary species to compare indirect vs. direct predation results.
- Results were analyzed accordingly and future populations were predicted.

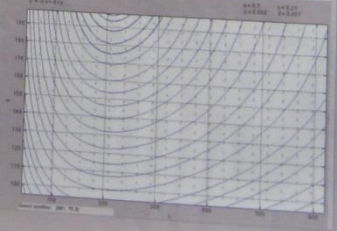
Table 1: Possum Population Data in the Wellington Region

Year	Estimated number of Possums	Estimated number of Tui birds
2000	15,000	265
2001	15,000	264
2002	15,000	263
2003	15,000	262
2004	15,000	261
2005	15,000	260
2006	15,000	259
2007	15,000	258
2008	15,000	257
2009	15,000	256
2010	15,000	255
2011	15,000	254
2012	15,000	253
2013	15,000	252
2014	15,000	251
2015	15,000	250
2016	15,000	249
2017	15,000	248
2018	15,000	247
2019	15,000	246
2020	15,000	245
2021	15,000	244
2022	15,000	243
2023	15,000	242
2024	15,000	241
2025	15,000	240
2026	15,000	239
2027	15,000	238
2028	15,000	237
2029	15,000	236
2030	15,000	235

## Results



Y =	MA Tui Population	MA Possum Population
254.0000	156.0000	
315.4475	168.9556	
371.2287	182.2709	
398.8452	229.4347	
375.9876	275.5865	
311.3311	315.4739	
234.4410	335.8922	
171.2962	333.1798	
138.5328	313.5532	
100.9333	285.3374	
95.5173	256.1584	
93.8093	228.8233	
98.5084	203.2077	
111.8180	182.8176	
132.1991	167.2341	
164.3512	151.4781	
207.9835	153.1954	
263.4681	157.8438	
325.3359	178.8596	
378.4528	197.2446	
398.6476	226.9648	
146.3333333	267.6666667	265
130.3333333		261.6666667
125.3333333		252.3333333
133.5		232.3333333
153.5		199
168.1666667		143.3333333
172.6666667		121.3333333
172.6666667		103
166.3333333		105.6666667
163		84.6666667
162.8333333		80.3333333
158.5		116.6666667
145.1666667		156.3333333
126.6666667		244.3333333
120		322.3333333
114.3333333		394.3333333
118.5		448.6666667



## Discussion and Conclusion

Based on a 4-Year Moving Average the data for the following year would be approximately 449 Tui birds, and 11,900 possums. The difference between the Lotka-Volterra Model and the moving average is evident. The error lies within the calculation for the Lotka-Volterra model. Finding the optimized parameters for the set of ODEs proved to be a larger challenge than anticipated. Given the computing limitations of Matlab R2014a simulations were often exited before completion. This kept error from being minimized and thus parameters from being optimized.

Even though the results did not yield intended results, it is clear that there is a relationship between the tui and possum. It can also be concluded that despite the possum and tui populations being indirectly related it would appear that the direct comparison between the two is adequate to measure their relationship. As the possum population increases the tui population suffers. With the help of possum control laws the possum population has decreased during which time an increase in tui population was seen.

Further research might be conducted using a different programming agent, such as VBA, to complete a simulation comparison between the sets of population data.



# Pro-Sociability in the TCNJ Community

Sahnaz Saleem  
Advisor: Professor Thayer

## Research Questions

- What patterns are there in how the responses are ranked?
- How do attitudes change when asked about other TCNJ students?
- How does a participant's attitude change from when they observe violence versus when they hear about violence?
- How do demographics play a role?

## Methods

The primary tools of analysis in this study were SAS and Microsoft Excel. A lot of the investigator dealt with using SAS code to manipulate the data into different forms to perform analysis in Excel and SAS. Two important SAS techniques used were Association Rule and Sequential Analysis. The first method creates "rules" based on the participant responses. Sequential Analysis also takes into account the order of the rankings.

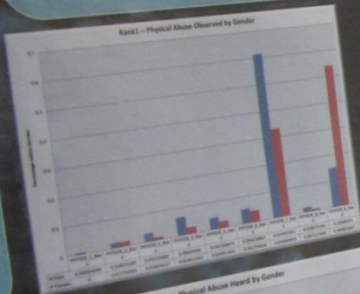
## Association Rule and Sequential Analysis

Support =  $P(A \cap B)$  - Support denotes the percentage of people who chose all of the answers present in the rule.  
Confidence =  $P(B|A) = \frac{P(A \cap B)}{P(A)}$  - Represents the respondents who also chose the right hand response while having chosen the left hand response.  
Lift =  $\frac{P(A \cap B)}{P(A) \cdot P(B)}$  - Measure of association between left hand response and right hand response. Values that are less than one represent a negative association, values higher than one represent positive association and values equal to one denote independence.

## Conclusions

- Overall TCNJ students are fairly pro-social.
- 53% of respondents scored pro-social across all 9 dimensions of violence.
- People are most pro-social when they observe the various situations when compared to hearing about them.
- Questions relating with "bearing" are being answered so students may have responded differently if they were given a time frame in which they heard about the incident.
- There is a clear distinction in what TCNJ students think about themselves and what they think of other TCNJ students.
- Respondents believed that their personal actions would be most pro-social than those of a typical TCNJ student.
- The respondents of the survey did not see any connecting behavior and stating a big of down in the other dimensions.
- Important to remember that these are preliminary conclusions and more research is needed to be conducted into more severe forms of violence.
- As a result, TCNJ students are not all violent and that violence and abuse comes in all shapes of forms and that any type deserves their attention and intervention.

Rank 1 - Physical Abuse Observed by Gender



Association Rule

Left Hand	Right Hand	Support	Confidence	Lift
Using a Weapon	Threatening	100%	100%	1.00
Using a Weapon	Kicking	100%	100%	1.00
Using a Weapon	Hitting	100%	100%	1.00
Using a Weapon	Biting	100%	100%	1.00
Using a Weapon	Other TCNJ Students	100%	100%	1.00
Threatening	Using a Weapon	100%	100%	1.00
Threatening	Kicking	100%	100%	1.00
Threatening	Hitting	100%	100%	1.00
Threatening	Biting	100%	100%	1.00
Threatening	Other TCNJ Students	100%	100%	1.00
Kicking	Using a Weapon	100%	100%	1.00
Kicking	Threatening	100%	100%	1.00
Kicking	Hitting	100%	100%	1.00
Kicking	Biting	100%	100%	1.00
Kicking	Other TCNJ Students	100%	100%	1.00
Hitting	Using a Weapon	100%	100%	1.00
Hitting	Threatening	100%	100%	1.00
Hitting	Kicking	100%	100%	1.00
Hitting	Biting	100%	100%	1.00
Hitting	Other TCNJ Students	100%	100%	1.00
Biting	Using a Weapon	100%	100%	1.00
Biting	Threatening	100%	100%	1.00
Biting	Kicking	100%	100%	1.00
Biting	Hitting	100%	100%	1.00
Biting	Other TCNJ Students	100%	100%	1.00
Other TCNJ Students	Using a Weapon	100%	100%	1.00
Other TCNJ Students	Threatening	100%	100%	1.00
Other TCNJ Students	Kicking	100%	100%	1.00
Other TCNJ Students	Hitting	100%	100%	1.00
Other TCNJ Students	Biting	100%	100%	1.00

Sequential Analysis

Left Hand	Right Hand	Support	Confidence	Lift
Using a Weapon	Threatening	100%	100%	1.00
Using a Weapon	Kicking	100%	100%	1.00
Using a Weapon	Hitting	100%	100%	1.00
Using a Weapon	Biting	100%	100%	1.00
Using a Weapon	Other TCNJ Students	100%	100%	1.00
Threatening	Using a Weapon	100%	100%	1.00
Threatening	Kicking	100%	100%	1.00
Threatening	Hitting	100%	100%	1.00
Threatening	Biting	100%	100%	1.00
Threatening	Other TCNJ Students	100%	100%	1.00
Kicking	Using a Weapon	100%	100%	1.00
Kicking	Threatening	100%	100%	1.00
Kicking	Hitting	100%	100%	1.00
Kicking	Biting	100%	100%	1.00
Kicking	Other TCNJ Students	100%	100%	1.00
Hitting	Using a Weapon	100%	100%	1.00
Hitting	Threatening	100%	100%	1.00
Hitting	Kicking	100%	100%	1.00
Hitting	Biting	100%	100%	1.00
Hitting	Other TCNJ Students	100%	100%	1.00
Biting	Using a Weapon	100%	100%	1.00
Biting	Threatening	100%	100%	1.00
Biting	Kicking	100%	100%	1.00
Biting	Hitting	100%	100%	1.00
Biting	Other TCNJ Students	100%	100%	1.00
Other TCNJ Students	Using a Weapon	100%	100%	1.00
Other TCNJ Students	Threatening	100%	100%	1.00
Other TCNJ Students	Kicking	100%	100%	1.00
Other TCNJ Students	Hitting	100%	100%	1.00
Other TCNJ Students	Biting	100%	100%	1.00

Association Rule

Left Hand	Right Hand	Support	Confidence	Lift
Using a Weapon	Threatening	100%	100%	1.00
Using a Weapon	Kicking	100%	100%	1.00
Using a Weapon	Hitting	100%	100%	1.00
Using a Weapon	Biting	100%	100%	1.00
Using a Weapon	Other TCNJ Students	100%	100%	1.00
Threatening	Using a Weapon	100%	100%	1.00
Threatening	Kicking	100%	100%	1.00
Threatening	Hitting	100%	100%	1.00
Threatening	Biting	100%	100%	1.00
Threatening	Other TCNJ Students	100%	100%	1.00
Kicking	Using a Weapon	100%	100%	1.00
Kicking	Threatening	100%	100%	1.00
Kicking	Hitting	100%	100%	1.00
Kicking	Biting	100%	100%	1.00
Kicking	Other TCNJ Students	100%	100%	1.00
Hitting	Using a Weapon	100%	100%	1.00
Hitting	Threatening	100%	100%	1.00
Hitting	Kicking	100%	100%	1.00
Hitting	Biting	100%	100%	1.00
Hitting	Other TCNJ Students	100%	100%	1.00
Biting	Using a Weapon	100%	100%	1.00
Biting	Threatening	100%	100%	1.00
Biting	Kicking	100%	100%	1.00
Biting	Hitting	100%	100%	1.00
Biting	Other TCNJ Students	100%	100%	1.00
Other TCNJ Students	Using a Weapon	100%	100%	1.00
Other TCNJ Students	Threatening	100%	100%	1.00
Other TCNJ Students	Kicking	100%	100%	1.00
Other TCNJ Students	Hitting	100%	100%	1.00
Other TCNJ Students	Biting	100%	100%	1.00

Sequential Analysis

Left Hand	Right Hand	Support	Confidence	Lift
Using a Weapon	Threatening	100%	100%	1.00
Using a Weapon	Kicking	100%	100%	1.00
Using a Weapon	Hitting	100%	100%	1.00
Using a Weapon	Biting	100%	100%	1.00
Using a Weapon	Other TCNJ Students	100%	100%	1.00
Threatening	Using a Weapon	100%	100%	1.00
Threatening	Kicking	100%	100%	1.00
Threatening	Hitting	100%	100%	1.00
Threatening	Biting	100%	100%	1.00
Threatening	Other TCNJ Students	100%	100%	1.00
Kicking	Using a Weapon	100%	100%	1.00
Kicking	Threatening	100%	100%	1.00
Kicking	Hitting	100%	100%	1.00
Kicking	Biting	100%	100%	1.00
Kicking	Other TCNJ Students	100%	100%	1.00
Hitting	Using a Weapon	100%	100%	1.00
Hitting	Threatening	100%	100%	1.00
Hitting	Kicking	100%	100%	1.00
Hitting	Biting	100%	100%	1.00
Hitting	Other TCNJ Students	100%	100%	1.00
Biting	Using a Weapon	100%	100%	1.00
Biting	Threatening	100%	100%	1.00
Biting	Kicking	100%	100%	1.00
Biting	Hitting	100%	100%	1.00
Biting	Other TCNJ Students	100%	100%	1.00
Other TCNJ Students	Using a Weapon	100%	100%	1.00
Other TCNJ Students	Threatening	100%	100%	1.00
Other TCNJ Students	Kicking	100%	100%	1.00
Other TCNJ Students	Hitting	100%	100%	1.00
Other TCNJ Students	Biting	100%	100%	1.00

Association Rule

Left Hand	Right Hand	Support	Confidence	Lift
Using a Weapon	Threatening	100%	100%	1.00
Using a Weapon	Kicking	100%	100%	1.00
Using a Weapon	Hitting	100%	100%	1.00
Using a Weapon	Biting	100%	100%	1.00
Using a Weapon	Other TCNJ Students	100%	100%	1.00
Threatening	Using a Weapon	100%	100%	1.00
Threatening	Kicking	100%	100%	1.00
Threatening	Hitting	100%	100%	1.00
Threatening	Biting	100%	100%	1.00
Threatening	Other TCNJ Students	100%	100%	1.00
Kicking	Using a Weapon	100%	100%	1.00
Kicking	Threatening	100%	100%	1.00
Kicking	Hitting	100%	100%	1.00
Kicking	Biting	100%	100%	1.00
Kicking	Other TCNJ Students	100%	100%	1.00
Hitting	Using a Weapon	100%	100%	1.00
Hitting	Threatening	100%	100%	1.00
Hitting	Kicking	100%	100%	1.00
Hitting	Biting	100%	100%	1.00
Hitting	Other TCNJ Students	100%	100%	1.00
Biting	Using a Weapon	100%	100%	1.00
Biting	Threatening	100%	100%	1.00
Biting	Kicking	100%	100%	1.00
Biting	Hitting	100%	100%	1.00
Biting	Other TCNJ Students	100%	100%	1.00
Other TCNJ Students	Using a Weapon	100%	100%	1.00
Other TCNJ Students	Threatening	100%	100%	1.00
Other TCNJ Students	Kicking	100%	100%	1.00
Other TCNJ Students	Hitting	100%	100%	1.00
Other TCNJ Students	Biting	100%	100%	1.00

Sequential Analysis

Left Hand	Right Hand	Support	Confidence	Lift
Using a Weapon	Threatening	100%	100%	1.00
Using a Weapon	Kicking	100%	100%	1.00
Using a Weapon	Hitting	100%	100%	1.00
Using a Weapon	Biting	100%	100%	1.00
Using a Weapon	Other TCNJ Students	100%	100%	1.00
Threatening	Using a Weapon	100%	100%	1.00
Threatening	Kicking	100%	100%	1.00
Threatening	Hitting	100%	100%	1.00
Threatening	Biting	100%	100%	1.00
Threatening	Other TCNJ Students	100%	100%	1.00
Kicking	Using a Weapon	100%	100%	1.00
Kicking	Threatening	100%	100%	1.00
Kicking	Hitting	100%	100%	1.00
Kicking	Biting	100%	100%	1.00
Kicking	Other TCNJ Students	100%	100%	1.00
Hitting	Using a Weapon	100%	100%	1.00
Hitting	Threatening	100%	100%	1.00
Hitting	Kicking	100%	100%	1.00
Hitting	Biting	100%	100%	1.00
Hitting	Other TCNJ Students	100%	100%	1.00
Biting	Using a Weapon	100%	100%	1.00
Biting	Threatening	100%	100%	1.00
Biting	Kicking	100%	100%	1.00
Biting	Hitting	100%	100%	1.00
Biting	Other TCNJ Students	100%	100%	1.00
Other TCNJ Students	Using a Weapon	100%	100%	1.00
Other TCNJ Students	Threatening	100%	100%	1.00
Other TCNJ Students	Kicking	100%	100%	1.00
Other TCNJ Students	Hitting	100%	100%	1.00
Other TCNJ Students	Biting	100%	100%	1.00

Situation: What would you do if you \_\_\_\_\_ your friend's partner hitting, kicking, biting, physically restraining your friend and/or threatening to use a weapon against your friend?

## to be Pro-Social?

It is important to understand "pro-social" and power based steps in intervening when others only take action when others are pro-social.

## Analysis

Very long and very complicated. I presented with a set of 15 questions in which presented with a situation and 9 perspectives: what would they do if in a situation, what would they do if at the situation, and what they TCNJ students would do when in a situation. The participants had to choose choices for each question. Five dimensions were addressed in every dimension are listed below.

## of Violence:



# Moduli Spaces of Triangles Inside Triangles

Author: Dan Seminara Advisor: Andrew Clifford

The College of New Jersey Department of Mathematics and Statistics

## Abstract

This project explores the idea of creating moduli spaces from properties of existing triangles. Cevians will be used to create these spaces and they will be compared to established moduli spaces of triangles. The main goal of this project is to show the thought process behind creating these spaces and to prove that moduli spaces composed of Almost Cevian triangles of isosceles triangles will only contain equilateral triangles if they are also acute.

## Moduli Spaces

Used to represent objects  
Objects need to be parameterized  
Define equivalence and modulation

## Cevian Triangles

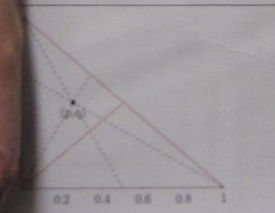
Cevians are the red dashed lines  
Cevian point is the red dot  
Cevian triangle is formed by solid red lines

## Almost Cevian Triangles

- Use the Cevians themselves to create triangles
- Doesn't always create a triangle (addressed later)
- Any point not on a vertex can be a Cevian Point

## Isosceles Right Triangle

Use blue dashed lines shown to create a triangle  
A moduli space based on side lengths will be used



## Side Length Formulas for AC-triangles Created from the Isosceles Right Triangle

• Cevian traveling through  $(0,0)$  and  $(p,q) \Rightarrow$

$$\frac{1}{\sqrt{2}}\sqrt{p^2+q^2}$$

• Cevian traveling through  $(0,1)$  and  $(p,q) \Rightarrow$

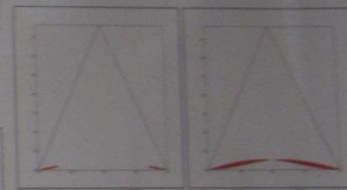
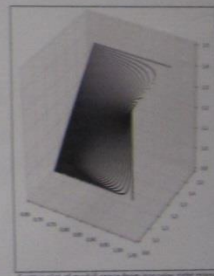
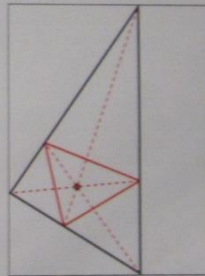
$$\frac{1}{\sqrt{2}}\sqrt{(p-1)^2+q^2}$$

• Cevian traveling through  $(1,0)$  and  $(p,q) \Rightarrow$

$$\frac{1}{\sqrt{2}}\sqrt{p^2+(q-1)^2}$$

## Questions

- Are all triangles represented in this new space?
- Is every point in this new space a triangle?



Points in red area create invalid triangles

## Almost Cevian Triangles from Isosceles Triangles

- Any isosceles triangle with side lengths  $a, b, c$  will only contain equilateral triangles in its moduli space of Almost Cevian triangles if it is acute.
- If the triangle is obtuse or right, then the range of AC side lengths are  $a$  to  $c$ ,  $a$  to  $c$ , and  $\sqrt{4a^2 - c^2}$  to  $a$
- Otherwise the ranges are  $\frac{c}{2}\sqrt{4a^2 - c^2}$  to  $c$ ,  $\frac{b}{2}\sqrt{4a^2 - c^2}$  to  $c$ , and  $\sqrt{4a^2 - c^2}$  to  $a$
- In the first case, the lower limit of the two equal sides is equal to the upper limit of the last side.
- The only location where all three limits can occur at once is on a vertex, but vertices are off-limits
- When the triangle is acute,  $2a^2 > c^2$  and this results in  $\frac{c}{2}\sqrt{4a^2 - c^2} < a$ . Since the ranges now overlap on more than just the boundary, equilateral triangles will exist in the moduli space

## References

- "Cevian Triangle." Wolfram MathWorld. N.p., n.d. Web. 13 Apr. 2014. <<http://mathworld.wolfram.com/CevianTriangle.html>>
- "Introduction." Matplotlib 1.3.1 documentation. N.p., n.d. Web. 14 Apr. 2014. <<http://matplotlib.org/1.3.1/index.html>>

# A FUNCTION ON THE MODULI SPACE OF TRIANGLES

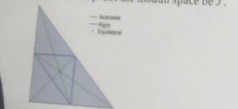
BY KATIE ROSE

## ABSTRACT

The moduli space of triangles is a tool we can use to look at all triangles at once. If we have a means to construct a new triangle out of an old one, we can view these constructions as functions on the moduli space of triangles.

## THE MODULI SPACE OF TRIANGLES

If the angles of a triangle be  $\alpha, \beta,$  and  $\gamma,$  where  $\gamma = 180 - \alpha - \beta.$  Then any triangle based on  $\alpha$  and  $\beta.$  Let the moduli space be  $\mathcal{T}.$



The limited space be  $\epsilon.$  Then  $\epsilon$  contains each triangle



## CONSTRUCTION

Extend two side lengths and bisect the new angles. The line formed is our constructed triangle. Let the new angles



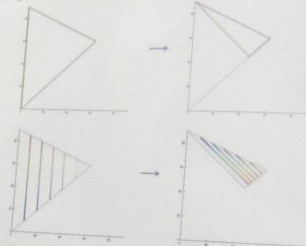
$$\left( \frac{\alpha}{2}, 90 - \frac{\alpha}{2}, 90 - \frac{\alpha}{2} \right)$$

$$\left( \frac{\beta}{2}, 90 - \frac{\beta}{2}, 90 - \frac{\beta}{2} \right)$$

$$\left( \frac{\gamma}{2}, 90 - \frac{\gamma}{2}, 90 - \frac{\gamma}{2} \right)$$

## THE FUNCTION ON $\epsilon$

The image is the set of acute triangles



## FIXED POINTS AND INVARIANT SETS

Use the inverse function.

### Fixed Points

$$\alpha = z = 180 - 2z \rightarrow 3z = 180 \rightarrow z = 60$$

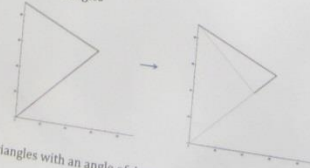
$$\beta = y = 180 - 2y \rightarrow 3y = 180 \rightarrow y = 60$$

$$\gamma = x = 180 - 2x \rightarrow 3x = 180 \rightarrow x = 60$$

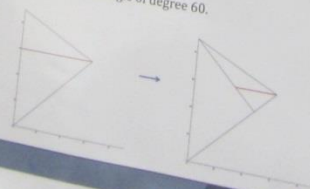
The only fixed point is the equilateral triangle.

### Invariant Sets

Isosceles Triangles



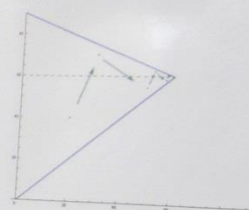
Triangles with an angle of degree 60.



## A SEQUENCE OF ITERATION

Let  $f_n(\alpha, \beta, \gamma)$  be the  $n^{\text{th}}$  iteration of  $f(\alpha, \beta, \gamma),$  or the  $n^{\text{th}}$  term in the sequence.

On  $\epsilon:$



From original construction



## LIMITING BEHAVIOR

**Theorem:** This sequence approaches the equilateral triangle.

**Proof:**

$f_n(\alpha, \beta, \gamma):$

$$x_n = (-1)^{n-1} \frac{180(2^{n-1} - 2^{n-2} + 2^{n-3} - \dots \pm 1) + [\alpha \text{ or } \gamma]}{2^n}$$

$$y_n = (-1)^{n-1} \frac{180(2^{n-1} - 2^{n-2} + 2^{n-3} - \dots \pm 1) + \beta}{2^n}$$

$$z_n = (-1)^{n-1} \frac{180(2^{n-1} - 2^{n-2} + 2^{n-3} - \dots \pm 1) + [\gamma \text{ or } \alpha]}{2^n}$$

where  $x_n$  is in terms of  $\alpha$  iff  $n$  is odd and  $z_n$  is in terms of  $\gamma$  iff  $n$  is even.

Take the limit as  $n$  goes to infinity of  $x_n, y_n,$  and  $z_n.$

## RESOURCES

Ben-Zvi, David D. "Moduli Spaces." *The Princeton Companion to Mathematics*. Ed. Timothy Gowers, June Barrow-Green, Imre Leader. Princeton: Princeton University Press, 2008. 420. Print.

Nakamura, H., K. Oguiso. "Elementary Moduli Space of Triangles and Iterative Processes." *Journal of Mathematical Sciences, The University of Tokyo* 10.1 (2003): 209-224. Electronic.

Nicollier, Grégoire. "Dynamics of the Nested Triangles Formed by the Tops of the Perpendicular Bisectors." *Forum Geometricorum* 14 (2014): 31-41. Electronic.

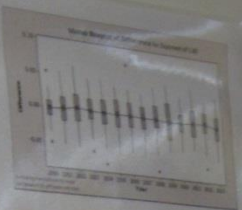
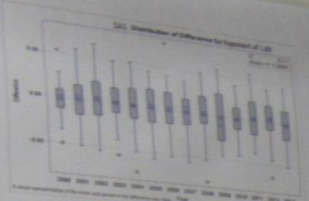
Rosenberg, Steven, Michael Spillane, Daniel Wulf. "Heron Triangles and Moduli Spaces." *Mathematics Teacher* 101.9 (2008): 656-562. Print.

Accuracy of the Pythagorean Winning Percentage  
 By: Matt Rusay  
 Advisors: Professor Thayer and Dr. Ochs

**Introduction**  
 I calculated 30 years worth of data and developed a formula called the Pythagorean Winning Percentage originally was:

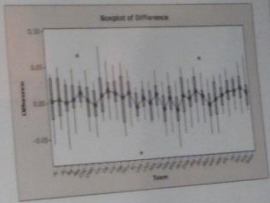
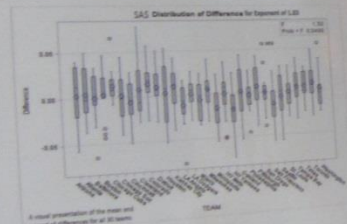
$$\text{Pythagorean Winning Percentage} = \frac{\text{runs scored}^2}{\text{runs scored}^2 + \text{runs allowed}^2} \times 9$$

Exponent of 2 was changed to an exponent of 1.83 by statisticians for accuracy. The next way to determine if this new model was a better predictor of what teams have done is to compare it to the actual winning percentages. Then, I found the difference between the two and used a regression to see if the difference was significant. Then, I used the difference between the two to see if a different result is generated between software or if the model is accurate.



Summary of Stepwise Selection

Step	Delta R Squared	F Value	Pr > F
1	0.000000	0.000000	1.000000
2	0.000000	0.000000	1.000000
3	0.000000	0.000000	1.000000
4	0.000000	0.000000	1.000000
5	0.000000	0.000000	1.000000
6	0.000000	0.000000	1.000000
7	0.000000	0.000000	1.000000
8	0.000000	0.000000	1.000000
9	0.000000	0.000000	1.000000
10	0.000000	0.000000	1.000000
11	0.000000	0.000000	1.000000
12	0.000000	0.000000	1.000000
13	0.000000	0.000000	1.000000
14	0.000000	0.000000	1.000000
15	0.000000	0.000000	1.000000
16	0.000000	0.000000	1.000000
17	0.000000	0.000000	1.000000
18	0.000000	0.000000	1.000000
19	0.000000	0.000000	1.000000
20	0.000000	0.000000	1.000000
21	0.000000	0.000000	1.000000
22	0.000000	0.000000	1.000000
23	0.000000	0.000000	1.000000
24	0.000000	0.000000	1.000000
25	0.000000	0.000000	1.000000
26	0.000000	0.000000	1.000000
27	0.000000	0.000000	1.000000
28	0.000000	0.000000	1.000000
29	0.000000	0.000000	1.000000
30	0.000000	0.000000	1.000000

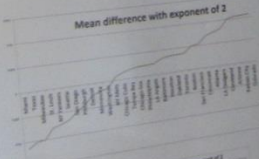
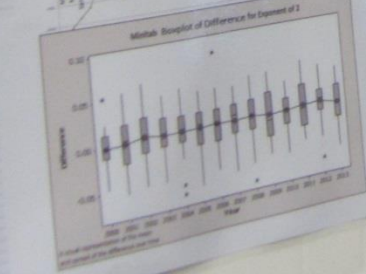


Normal Boxplot of Difference for Exponent of 1.83

Statistic	Value
Minimum	-0.000000
Q1	-0.000000
Median	0.000000
Q3	0.000000
Maximum	0.000000

Summary of Stepwise Selection

Step	Delta R Squared	F Value	Pr > F
1	0.000000	0.000000	1.000000
2	0.000000	0.000000	1.000000
3	0.000000	0.000000	1.000000
4	0.000000	0.000000	1.000000
5	0.000000	0.000000	1.000000
6	0.000000	0.000000	1.000000
7	0.000000	0.000000	1.000000
8	0.000000	0.000000	1.000000
9	0.000000	0.000000	1.000000
10	0.000000	0.000000	1.000000
11	0.000000	0.000000	1.000000
12	0.000000	0.000000	1.000000
13	0.000000	0.000000	1.000000
14	0.000000	0.000000	1.000000
15	0.000000	0.000000	1.000000
16	0.000000	0.000000	1.000000
17	0.000000	0.000000	1.000000
18	0.000000	0.000000	1.000000
19	0.000000	0.000000	1.000000
20	0.000000	0.000000	1.000000
21	0.000000	0.000000	1.000000
22	0.000000	0.000000	1.000000
23	0.000000	0.000000	1.000000
24	0.000000	0.000000	1.000000
25	0.000000	0.000000	1.000000
26	0.000000	0.000000	1.000000
27	0.000000	0.000000	1.000000
28	0.000000	0.000000	1.000000
29	0.000000	0.000000	1.000000
30	0.000000	0.000000	1.000000



Normal Boxplot of Difference for Exponent of 2

Statistic	Value
Minimum	-0.000000
Q1	-0.000000
Median	0.000000
Q3	0.000000
Maximum	0.000000

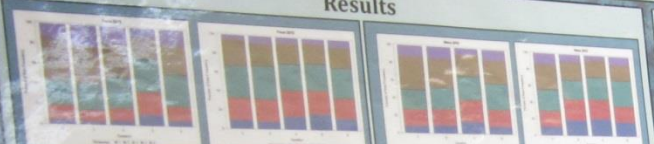


**Conclusion**  
 The Pythagorean winning percentage is a good predictor of what teams have done. The difference between the two is not significant. The difference between the two is not significant. The difference between the two is not significant.

# TCNJ Dining Services: Are Students Satisfied?

**Caitlin Stack**  
 Advisor: Richard Thayer  
 The College of New Jersey: Department of Mathematics and Statistics

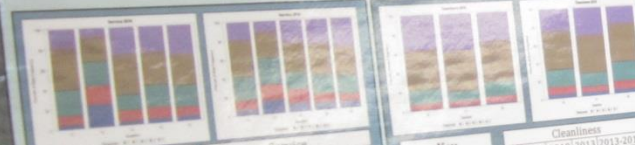
## Results



Key	Question	2010	2013	2010-2013
2: Taste		3.44621	2.95691	-0.50931
3: Eye Appeal		3.24131	3	-0.6143
4: Freshness		4.34611	2.78954	-1.67555
5: Nutritional Content		5.29539	3.2741	-2.02989
6: Value		4.33541	2.9075	-1.42791

### Conclusions by Category

- Students are satisfied with Eickhoff Hall.
- However, students are less satisfied with Eickhoff in 2013 than 2010.
- Most categories had a decrease in mean response and few remained similar.
- The overall mean response decreased from 3.68 in 2010 to 3.45 in 2013 (out of 5).
- The healthiness of the food and the size of Eickhoff relative to the size of student body at TCNJ can be most improved.



Key	Question	2010	2013	2010-2013
11: Overall		3.88881	3.64961	-0.23921
12: Speed of Service		3.29721	3.84761	0.55041
13: Hours of Operation		3.97411	3.86761	-0.10651
14: Helpfulness of Staff		4.68421	3.85761	-0.82661
15: Friendliness of Staff		3.72781	3.71921	-0.00861

### Advice to Client: Category

- Healthiness of the Food: less satisfied with 'Nutritional Content', 'Freshness', 'Variety of Healthy Menu Options'. Offer more and healthier options at Eickhoff Hall.
- Size: less satisfied with 'Speed of Service', 'Cleanliness of Eating Areas', and 'Availability of Seating'. Eickhoff Hall needs to expand in relation to the growing student body.



Key	Question	2010	2013	2010-2013
17: Location		3.97761	3.67681	-0.30081
18: Layout		3.62761	3.68761	0.06001
19: Appearance		4.05761	3.68761	-0.37001
20: Avail. of Seating		4.05761	3.68761	-0.37001
21: Comfort		3.72781	3.71921	-0.00861

### Conclusions by Demographic

- Freshman are the most satisfied.
- Seniors are the least satisfied.
- No difference in satisfaction between gender.
- Off campus tends to be more satisfied than on campus.



Key	Demographic	2010	2013	2010-2013
1: Freshman		3.97761	3.67681	-0.30081
2: Sophomore		3.62761	3.68761	0.06001
3: Junior		4.05761	3.68761	-0.37001
4: Senior		4.05761	3.68761	-0.37001

### Advice to Client: Demographic

- Student satisfaction decreases as with age.
- Theme Nights, 'Senior' discounts, etc. to keep the older students satisfied and going to dining locations on campus.



Key	Demographic	2010	2013	2010-2013
1: On Campus		3.97761	3.67681	-0.30081
2: Off Campus		3.62761	3.68761	0.06001

### Advice to Client: Overall

- Better raw data = better results.
- Use an incentive to get more data for other locations and respondents.
- TCNJ Dining Services could create their own survey to better suit the needs of TCNJ.



**Objective**  
 To analyze the raw data  
 Customer Satisfaction

...tion levels between  
 ...nt questions by category  
 ...etc.) to find what  
 ...st and least satisfied

...graphics of the survey  
 ...r these have an effect on  
 ... find the most satisfied  
 ...us.

**Background Information**  
 Sodexo is a French  
 company that employs  
 people in over 80 different

Association of College and  
 Food Service (NACUFS) was  
 established in 1958 to promote and  
 the highest quality food  
 campus.

**ANOVA Testing**  
 ANOVA test used to compare means  
 question or demographics.  
 $y = \mu + \alpha_i + \epsilon(0, \sigma^2)$   
 instead of doing multiple t-tests  
 with a very high error.  
 Assumptions: Independent, random  
 and  $\epsilon(0, \sigma^2)$ .

**Raw Data**  
 ... with small sample size in  
 ... other than Eickhoff Hall  
 ... in 2010 and 68.08% in 2013).  
 ... with small sample size in the  
 ... other than students  
 ... 2010 and 79.24% in 2013).  
 ... students' reviewing

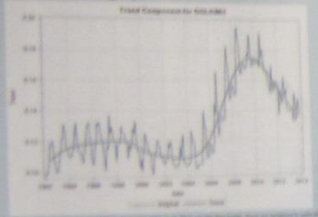
# Exploring monthly electric bills for an all-electric home

Roger Shan, Dr. Richard Thayer, The College of New Jersey

## Introduction

Monthly electric bills display the amount of electricity used and the rates charged for producing and delivering it. Objectives

1. Identify major rate changes by the Utility and seasonal patterns
2. Examine usage before and after the installation of a new heating system.

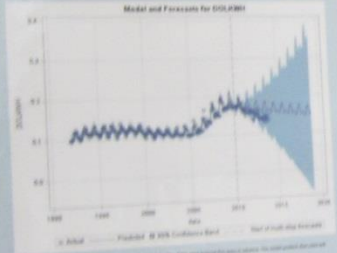


## Method and Results: Rates Analysis

Smooth data to obtain seasonally and trend components. In SAS, PROC ESM implements exponential smoothing. The Holt-Winters method was used because the series has seasonality and trend.

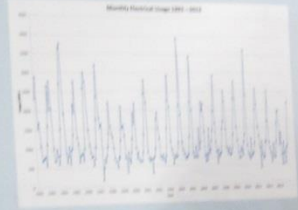
$$\begin{aligned}
 \hat{Y}_t &= \alpha Y_t + (1 - \alpha) \hat{Y}_{t-1} \\
 Y_t &= \beta(Y_t - Y_{t-1}) + (1 - \beta)Y_{t-1} \\
 I_t &= \alpha(Y_t - I_{t-1}) + (1 - \alpha)I_{t-1} \\
 S_t &= (1 - \gamma)S_t + \gamma Y_{t-1}
 \end{aligned}$$

- estimate of seasonal multiplicative factor
- estimate of non-seasonal trend
- estimate of basic level
- $\alpha, \beta, \gamma$  smoothing constants with values between 0 and 1, chosen by SAS
- $I_t$  forecast based on level, trend, and seasonality estimates



## Method and Results: Usage Analysis

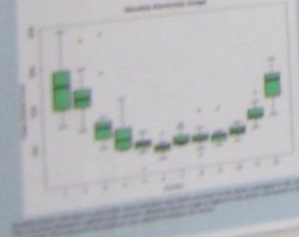
A new heating system was installed in 1996. Was it more efficient than the previous system? The plot shows that electricity use is fairly consistent about a fixed level. There is no trend. There is a visible seasonal component.



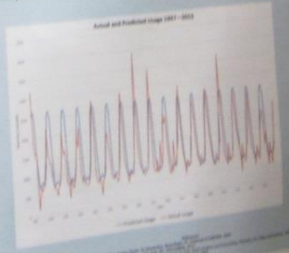
Data from 1992—1996 were used to predict usage for 1997—2013. The model consistently over-predicted usage in the winter months, which suggests that the new heating system was more efficient than the one it replaced. The homeowner realized a savings of about \$6300 from 1997 to 2013. Much of the savings occurred during the winter months, where the new heating system outperformed the previous one.

## First Steps

Visualize data with boxplots and line graphs. Identify median monthly rates and usage. Look for seasonal patterns and trends.



In 1999, NJ Gov. Christine Whitman deregulated utilities in New Jersey. Residential customers received 5% discounts on their electric bills. The discounts remained in effect from 1999 to 2003. Rates from 1999—2003 were used to forecast rates from 2004—2013. The model consistently predicted lower rates.





# Chaos in the Cantor Set

Jamie Warren  
The College of New Jersey  
Advisor: Andrew Clifford  
Spring 2014

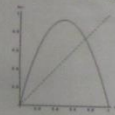
## Abstract

The purpose of this project is to create two connections between the Cantor set, a set that is easily understood by most mathematicians, students, and chaos theory, field of mathematics that is still being developed. This is helpful because understanding this connection could make the study of chaos theory more exciting, which could potentially lead to a higher interest in pursuing research on this topic.

## Two Views of the Cantor Set



## Logistic Equation

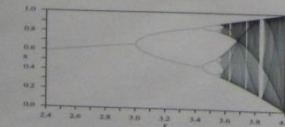


$$f_r(x) = rx(1-x)$$

When  $1 < r < 3$  we have an attractor

$$\begin{aligned} f_2(0.2) &= 2(0.2)(1-0.2) = 0.32 \\ f_2(0.32) &= 2(0.32)(1-0.32) = 0.4352 \\ f_2(0.4352) &= 2(0.4352)(1-0.4352) = 0.49160192 \\ f_2(0.49160192) &= 2(0.49160192)(1-0.49160192) = 0.4998589445 \\ f_2(0.4998589445) &= 2(0.4998589445)(1-0.4998589445) = 0.499999602 \\ \lim_{n \rightarrow \infty} (f_2^n(0.2)) &= 0.5 \end{aligned}$$

## Bifurcation

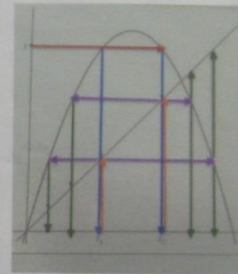


- When  $r > 3$ , single attractor splits into two
- $r = 3.4495$  four attractors
- $r = 3.56$  eight attractors
- $r = 3.569$  sixteen attractors
- Eventually becomes infinitely many, shown in Feigenbaum's diagram in the gray area

## Relationship Between the Two

- Produced by iterations
- Self-similar- smaller parts of the set look the same as the set as a whole

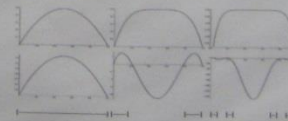
## Backwards Iteration



## First Connection

- Backwards iteration when  $r = 4.5$  gives us the Cantor Set, giving us our first connection between the two.
- This raises the question: Do any other values of  $r$  yield the Cantor set?

The first row is the first three backward iterations of the logistic equation when  $r = 2$ . The second row is the first three backward iterations of the logistic equation when  $r = 6$ .



We can see that the value when  $r = 6$  seems to produce values that look similar to that of the Cantor set

## Theorem

Given the logistic map  $f_r(x) = rx(1-x)$  let  $A_r = \bigcap_{n=0}^{\infty} f_r^{-n}([0,1])$  where  $f_r^{-n}$  is the backward iteration  $n$  times, if  $r \geq 6$  then  $A_r$  is a Cantor set

Suppose  $A_r$  contains the interval  $[a,b]$

The Mean Value Theorem gives us that  $\exists c_n \in (a,b)$  such that  $(f_r^n)'(c_n) = \frac{f_r^n(b) - f_r^n(a)}{b-a}$  which means  $(f_r^n)'(c_n)(b-a) = f_r^n(b) - f_r^n(a)$

Let  $q_n < q_i$ , we can solve  $f_r(x) = 1$  where  $l_1 = [0,1]$  and  $l_2 = [0, q_1] \cup [q_1, 1]$

Let  $\lambda = f_r'(q_n)$  so  $|f_r'(x)| \geq \lambda \forall x \in l_1$  and  $|\lambda| > 1$

We can say  $|(f_r^n)'(c_n)| \geq \lambda^n$  where  $k \leq n-1$  and plugging into our equation

we get  $|f_r^n(b) - f_r^n(a)| = |(f_r^n)'(c_n)|(b-a) \geq \lambda^n(b-a)$

$\therefore |f_r^n(b) - f_r^n(a)| > 1$

$\therefore [f_r^n(a), f_r^n(b)] \not\subset A_r$  and so  $f_r^n(a)$  and  $f_r^n(b) \notin [0,1]$

$\therefore |f_r^n(a) - f_r^n(b)| < 1$  contradiction

$\therefore A_r$  does not contain any intervals

$\therefore A_r$  is a Cantor set

Note: The exact value for  $r$  is when  $r > 2 + \sqrt{5}$  however we use  $r \geq 6$  for simplicity in this proof

## Second Connection

- Feigenbaum's number: ratio between steps 4.669:1
- Feigenbaum's point 4.669: chaos initially sets in
- Tips of the branches form the Cantor set

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# An Analysis Of Portfolio Allocation Strategies

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## Intro

Introductory text about portfolio allocation strategies.

## Strategic Asset Allocation

- Investor determines the proportion or amount of each type of asset class that they would like their long-term portfolio to be comprised of.
- Over a chosen period of time, such as annually or semi-annually, strategic asset allocation rebalances the portfolio to bring the proportions or amounts of asset classes back to their selected allocations.

## Tactical Asset Allocation

- Investors make short-term portfolio adjustments that change the portfolio mix between the different asset classes.
- If an asset class is forecasted to do well in the present, an investor will allocate a greater proportion of the portfolio to that asset class.
- Tactical asset allocation is a form of market timing.

## Example of Buy and Hold

- In a buy and hold strategy, an investor buys a stock and holds it for a long period of time, regardless of market fluctuations.
- The reallocation takes place based on the market.



## Risk

Text describing risk in portfolio allocation.

## Applied Method: Markowitz's Mean-Variance Model

- Attempts to find the portfolio that minimizes the risk for a given return according to modern portfolio theory.
- Standard deviation is the risk measurement used.
- To apply the model, the mean return of each stock must first be calculated.
- Next, a covariance matrix is set up between each of the different assets.
- The variance of each stock is then calculated.
- The total portfolio variance is minimized using the Mean-Variance model attempt to find the optimal portfolio return with the least amount of risk.
- The desired return is then found by allocating the portfolio according to the weights given from the model.

## Example of Mean-Variance Model (4 Year Allocations)

- I tested this allocation model every 4 years, beginning from the year 2000, up until 2014.
- So every 4 years, the allocation percentage of each stock changes in order to minimize risk.
- Consider investing \$100,000 in each of the 6 stocks, as well as the S&P500.
- Note that 2008 returns to the rate of return of all investments over a given period of time.

Year	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5	Stock 6	S&P500
2000	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2004	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2008	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2012	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%

## Example of Mean-Variance Model (2 Year Allocations)

- Next, the model was tested every 2 years, beginning from the year 2000, up until 2014.
- So every 2 years, the allocation percentage of each stock changes in order to minimize risk.
- The same process will repeat by investing \$100,000 in each of the 6 stocks, as well as the S&P500.

Year	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5	Stock 6	S&P500
2000	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2002	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2004	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2006	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2008	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2010	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2012	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%
2014	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%	16.7%

## Application

- In order to apply this portfolio allocation method, I selected 6 large cap stocks and the S&P500.
- This allocation method will be compared to a typical buy and hold strategy using 2 year allocations and 4 year allocations.
- Stocks chosen: AAPL, MSFT, GE, IBM, and AMZN, in order to pick a large cap company from different sectors of the economy.



## Conclusion

- In this particular case, the buy and hold strategy ended up with a return of 7.9%, the 4 year allocation model yielded a return of 7.7%, and the 2 year allocation model resulted in a return of 7.8%.
- The 2 year allocation model outperformed the buy and hold strategy by 2.54%, and beat the 4 year allocation model by 2.56%.

## Future Work

- Integrate the computer program on a dedicated system that able to reallocate securities in a portfolio on a real-time basis, conditions that would enable the optimal time to reallocate a portfolio.
- The core program would also need to include security that other than stocks, such as bonds and derivative securities.

# Automorphisms of Lie Groups

## Motivation

- What is the relationship between the automorphism group of a connected Lie group and its mapping class group?

## Key Concepts

### Definition

A Lie group is a group that is also a manifold, equipped, in some way, with the group operations of multiplication and inversion, which are smooth maps.

## The Mapping Class Group

### Definition

The mapping class group of a Lie group  $G$  is the group of equivalence classes of diffeomorphisms  $f: G \rightarrow G$  which are automorphisms of  $G$ .

The mapping class group of  $G$  is denoted by  $\mathcal{M}(G)$ .

## Question Revisited

- We are interested in the case where  $G$  is a connected Lie group.

$$\begin{array}{ccc} \text{Aut}(G) & \longrightarrow & \mathcal{M}(G) \\ \downarrow & & \downarrow \\ \text{Aut}(G) & \xrightarrow{\cong} & \text{Aut}(G) \end{array}$$

## It is Well-Defined



### Lemma

If  $f$  is an automorphism of a Lie group  $G$ , then  $[f]$  is well-defined.

$$\begin{aligned} (1) \quad [f] &= [f] \\ (2) \quad [f] &= [f] \\ (3) \quad [f] &= [f] \end{aligned}$$

## Conclusion

- $\mathcal{M}(G)$  is well-defined (every diffeomorphism is an automorphism).
- $\mathcal{M}(G)$  is well-defined (every diffeomorphism is an automorphism).

## The Exact

$$\begin{array}{ccc} \text{Aut}(G) & \longrightarrow & \mathcal{M}(G) \\ \downarrow & & \downarrow \\ \text{Aut}(G) & \xrightarrow{\cong} & \text{Aut}(G) \end{array}$$

## $\mathcal{M}(G)$

$$\begin{array}{ccc} \text{Aut}(G) & \longrightarrow & \mathcal{M}(G) \\ \downarrow & & \downarrow \\ \text{Aut}(G) & \xrightarrow{\cong} & \text{Aut}(G) \end{array}$$

## Conclusion

- $\mathcal{M}(G)$  is well-defined (every diffeomorphism is an automorphism).
- $\mathcal{M}(G)$  is well-defined (every diffeomorphism is an automorphism).

## References

Brooke F. James, "The Automorphism Group of a Lie Group," *Journal of Lie Theory and Related Structures*, 1998.



# Investigating Seasonality in Financial Data: Using Monthly Returns and Key Economic Dates

The College of New Jersey, Department of Mathematics  
David Algava

## Abstract

Seasonality in financial data has been a topic of interest for many years. This paper examines the relationship between monthly returns and key economic dates. The data is analyzed using a variety of statistical methods, including regression analysis and the Turkey's Comparison Test. The results suggest that there is a significant relationship between the two variables, and that the Turkey's Comparison Test is a useful tool for identifying seasonal patterns in financial data.

## Return on Investment

- The return on investment (ROI) is a measure of the profitability of an investment.
- It is calculated as the net profit divided by the initial investment.
- The formula for ROI is:  $ROI = \frac{\text{Net Profit}}{\text{Initial Investment}}$

## Tukey's Comparison Test on Monthly Returns

- Tukey's Comparison Test is a statistical test used to compare the means of two or more groups.
- It is based on the Tukey's range statistic, which is the difference between the largest and smallest values in a sample.
- The test is used to determine if there are significant differences between the groups.

## Sorting Monthly Returns

- Monthly returns are sorted in descending order.
- The top 10% of returns are compared to the bottom 10%.
- The difference between the two groups is used to determine if there is a significant difference.

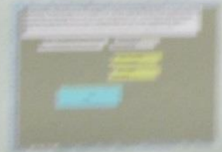


Figure 1: Results of the Turkey's Comparison Test. The chart shows the range statistic for each group, with significant differences indicated by asterisks.

## Tukey's Results

- The results of the Turkey's Comparison Test show significant differences between the groups.
- The test is used to determine if there are significant differences between the groups.

Group	Mean	Lower Bound	Upper Bound
Group 1	0.012	0.005	0.019
Group 2	0.008	0.001	0.015
Group 3	0.004	-0.003	0.007
Group 4	0.001	-0.006	0.004
Group 5	-0.002	-0.009	0.005
Group 6	-0.005	-0.012	0.002
Group 7	-0.008	-0.015	-0.001
Group 8	-0.011	-0.018	-0.004
Group 9	-0.014	-0.021	-0.007
Group 10	-0.017	-0.024	-0.010

## Summary of Tukey's Comparison Test

- The results of the Turkey's Comparison Test show significant differences between the groups.
- The test is used to determine if there are significant differences between the groups.

## Introduction

The purpose of this study is to investigate the relationship between monthly returns and key economic dates. The data is analyzed using a variety of statistical methods, including regression analysis and the Turkey's Comparison Test. The results suggest that there is a significant relationship between the two variables, and that the Turkey's Comparison Test is a useful tool for identifying seasonal patterns in financial data.

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# A Comparative Analysis of Forecasting US Energy Prices

Christopher Benvenuto, Faculty Advisor: Dr. Ed Conjura  
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## Abstract

This project involves the examination of energy prices under the categories of heating oil, natural gas, and electricity. Data on energy prices from 1995 to 2013 will be used in an attempt to forecast future values of these prices. This will be done utilizing an Excel Visual Basic program and SAS code, and forecasting results will be compared to existing programs. Computed forecast values will also be compared against the forecasted energy prices provided by the source of the data. Forecasting methods used in this project include Moving Average along with Single, Double, and Triple Exponential Smoothing. The best forecasting method for each type of energy price will be determined after comparing the mean absolute percentage errors of each method.

## Introduction

- Why forecast energy prices?
- Necessary in many fields of study
- Benefits both producers and consumers
- How to forecast future energy prices?
- Time Series Analysis
  - Methods
    - Moving Averages
    - Exponential Smoothing
    - Software
    - Excel Visual Basic
    - SAS
- Source of data for this project is the U.S. Energy Information Administration (eia)
- Range of data: 1995-2013
- Quarterly prices
- Data also includes forecasted prices for 2014 and 2015
- Generated by simulation of the EIA Regional Short-Term Energy Model

## Methods

Energy prices have been forecasted utilizing the following methods in Excel VBA and SAS using parameter values that will minimize the MAPE derived from Excel VBA program.

- 2-Period Moving Average
- Single Exponential Smoothing
  - Heating Oil
    - $\alpha = 0.95$
    - Natural Gas
      - $\alpha = 0.25$
    - Electricity
      - $\alpha = 0.3$
  - Double Exponential Smoothing
    - Heating Oil
      - $\alpha = 0.95, \beta = 0$
    - Natural Gas
      - $\alpha = 0.3, \beta = 0$
    - Electricity
      - $\alpha = 0.5, \beta = 0.1$
  - Triple Exponential Smoothing
    - Heating Oil
      - $\alpha = 0.5, \beta = 0.5, \gamma = 1.0$
    - Natural Gas
      - $\alpha = 0.3, \beta = 0, \gamma = 0.3$
    - Electricity
      - $\alpha = 0.3, \beta = 0, \gamma = 0.4$

## Results

VBA

Screen Capture of Excel VBA Program Input Worksheet

Screen Capture of Excel VBA Output for Heating Oil Prices

SAS

Screen Capture of SAS User Interface

Screen Capture of SAS Output for Heating Oil Moving Average Forecast

Heating Oil VBA All Methods

	MSD		MAD		MAPE	
	VBA	SAS	VBA	SAS	VBA	SAS
MA	0.014240	0.014740	0.072600	0.072600	3.46%	3.46%
ES	0.058968	0.058224	0.148339	0.147062	7.22%	6.95%
DES	0.058968	0.056837	0.148339	0.144773	7.22%	7.00%
TES	0.149913		0.240469		11.34%	

Natural Gas VBA All Methods

	MSD		MAD		MAPE	
	VBA	SAS	VBA	SAS	VBA	SAS
MA	1.479E-04	1.479E-04	0.0096367	0.0096367	8.68%	8.68%
ES	4.248E-04	4.248E-04	0.0166089	0.0166089	15.79%	15.79%
DES	4.265E-04	4.168E-04	0.0167963	0.0166047	15.89%	15.60%
TES	4.631E-04		0.0168825		15.36%	

Electricity VBA All Methods

	MSD		MAD		MAPE	
	VBA	SAS	VBA	SAS	VBA	SAS
MA	5.900E-06	5.900E-06	0.0020933	0.0020933	0.00386656	0.00386656
ES	2.225E-05	2.168E-05	0.0043700	0.0043503	0.0041425	0.0041425
DES	2.550E-05	2.526E-05	0.0043700	0.0043503	0.0041425	0.0041425
TES	2.768E-05					

SAS Graph for Heating Oil h

Comparative Analysis of Forecasting US Energy  
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# Exploring Seasonality in the Stock Market

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## Introduction

Many people invest in the stock market, but not many people know how to effectively predict the market. For the most part this is not ignorance but the fact that it is just difficult to predict the market, and even after getting a decent model the market is constantly changing and thus many of the model will be obsolete eventually. It all starts with that question, how can we better predict the market? Well that is where this project came in.

This project is being conducted to investigate a seasonal component in the market that can help us better predict future stock prices, and returns. Seasonality is any predictable change that occurs during any period of time. Seasonality within a year can be monthly, quarterly, bi-annually, etc. We can investigate the seasonal component of a time series through modeling or the use of other statistical methods to determine how seasonal, if there is any, is affecting stock prices and returns. Out of the many options this project will be using Tukey's Method, and a Holt-Winters Model.

Technology was a vital part in the execution of this project as many of the calculations and data manipulations are difficult, if not impossible, to do by hand. In particular, SAS was used for the modeling and statistical test, while Visual Basic was used to deal with sorting issues, and making calculations.

## Methods

### Tukey's Method

Invented by John Tukey  
Multiple comparison test which compares all pairs of means  
Similar to a t-test, but corrects for multiple testing  
Distribution (q)  
Test statistic can be calculated using the formula:

$$q_j = \frac{\bar{M}_i - \bar{M}_k}{SE}$$

where  $\bar{M}_i$  is the larger mean,  $\bar{M}_k$  is the smaller mean, and SE is the standard error.  
The test statistic is compared to the critical value using the Studentized Range Distribution  
It is used to compare monthly returns to see if any months have a statistically significant difference

## Methods con't

### Holt-Winters Model

- Developed by C.C. Holt and his student Peter Winters
- The Holt-Winters Model is a smoothing model that takes into account seasonality
- There is an additive and a multiplicative Holt-Winters Model
- They are both comprised of three equations that are used to get the forecasted values
- The three equations are the overall smoothing ( $E_t$ ), the trend estimation ( $T_t$ ), and the seasonal index ( $S_t$ ), along with the forecast ( $F_t$ )
- For the multiplicative model they are

$$E_t = \frac{Y_t}{S_{t-L}} + (1 - \alpha) * (E_{t-1} + T_{t-1})$$

$$T_t = \beta * (E_t - E_{t-1}) + (1 - \beta) * T_{t-1}$$

$$S_t = \gamma * \left(\frac{Y_t}{E_t}\right) + (1 - \gamma) * S_{t-L}$$

- Where  $\alpha$ ,  $\beta$ , and  $\gamma$  are constant weights between 0 and 1, t is time, and L is the length of the seasonal period
- When  $\alpha$ ,  $\beta$ , and  $\gamma$  are large there is more weight given to more recent observations
- The forecast for time t+M are given by  
 $F_{t+M} = (E_{t-1} + M * T_{t-1}) * S_{t+M-L}$
- Where M is how far into the future you want to forecast
- Using SAS, Holt-Winters will be used to compare market indexes with seasonality and without both with best observed parameters (Lowest root mean squared error)

## Results

Tukey's test with alpha = .10

INDEX	Significant Difference
S&P 500	September and December
Nikkei	August and December
Dow	March and May March and August August and October August and November
DAX	April and August April and September

## Results Con't

Best Observed Parameters for Winters Model with Seasonality

Index	Parameters	RMSE
S&P 500	$\alpha = .76$ $\beta = .05$ $\gamma = .44$	3.12
Dow	$\alpha = .76$ $\beta = .08$ $\gamma = .18$	26.1
NASDAQ	$\alpha = .80$ $\beta = .10$ $\gamma = .25$	126.1

Best Observed Parameters for Winters Model without Seasonality

Index	Parameters	RMSE
S&P 500	$\alpha = .90$ $\beta = .09$	28.70
Dow	$\alpha = .92$ $\beta = .06$	244.81
NASDAQ	$\alpha = .87$ $\beta = .05$	121.71

- Conclusion
- The Tukey's test indicates that in certain stock indexes there are months where the mean return is statistically different from another.
  - August and September have consistently lower returns than November and December have higher returns.
  - Suggests traders should be more cautious during these months leading to being skinnier.

and Future Work

ferences

knowledge

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# Hitting Vs. Pitching: The True Importance

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The College of New Jersey

## Abstract

What really causes a baseball team to win? Is it a matter of keeping runs off the scoreboard through pitching or putting them on the scoreboard by hitting? The goal of this research project was to definitively answer the question of whether pitching or hitting is more important to a team's success over the last several years. The goal of this research project was to definitively answer the question of whether pitching or hitting is more important to a team's success over the last several years.

## Introduction

America's favorite pastime has been around since the early 1800s; however, the rules and manner in which the game is played are very different today. Each team plays with 9 players arranged around a diamond (field) in various positions, as shown below in Figure 1. The goal of the field players is to get the batter out. Batters use a bat to hit a ball thrown by a pitcher and thus a pitcher tries to keep runs from scoring and a batter tries to score runs. Each team takes turns hitting and fielding. The goal of the game is to score more runs than the other team. Over the years a vast number of statistics have been developed to keep track of a team's performance on the field. Normally, the success or failure of a team can be seen in these various statistics. The data set used contained 45 team hitting and pitching variables at the beginning of the analysis. One of my main goals was to decrease the amount of variables used to predict win totals. This begs the question: can we find which of those statistics are most important to look at when determining a team's success or lack thereof? In order to determine this, I used a combination of random forest classification and linear regression. The resulting analysis and conclusions drawn from it are presented in this poster.



Figure 1

R = Run	SB = Stolen Bases	CF = Center Fielder
HR = Home Run	SO = Strike Outs	RF = Right Fielder
BB = Base on Balls	SH = Sacrifice Hits	2B = Second Base
W = Wins	SL = Strikeouts per 9 Inning Pitched	SS = Shortstop
ERA = Earned Run Average	SHO = Shutouts	3B = Third Base
WHIP = Walks and Hits per Inning Pitched	SV = Saves	P = Pitcher
OPS = On Base Percentage	IP = Inning Pitched	
SLA = Slugging Average	IPF = Inning Pitched Fraction	
OBP = On Base Percentage	IPF = Inning Pitched Fraction	
SLA = Slugging Average	IPF = Inning Pitched Fraction	
OBP = On Base Percentage	IPF = Inning Pitched Fraction	

## Methods

Team	2007	2008	2009
Ari	96.5	95.5	94.5
Atl	95.5	94.5	93.5
Bos	94.5	93.5	92.5
Br	93.5	92.5	91.5
Chi	92.5	91.5	90.5
Cin	91.5	90.5	89.5
Cle	90.5	89.5	88.5
Col	89.5	88.5	87.5
Det	88.5	87.5	86.5
Fla	87.5	86.5	85.5
Ind	86.5	85.5	84.5
Kc	85.5	84.5	83.5
Lad	84.5	83.5	82.5
Laa	83.5	82.5	81.5
Lgb	82.5	81.5	80.5
Mil	81.5	80.5	79.5
Mia	80.5	79.5	78.5
Mt	79.5	78.5	77.5
Nyn	78.5	77.5	76.5
Oak	77.5	76.5	75.5
Pit	76.5	75.5	74.5
Sf	75.5	74.5	73.5
Sea	74.5	73.5	72.5
Tex	73.5	72.5	71.5
Tor	72.5	71.5	70.5
Wsh	71.5	70.5	69.5
Wsn	70.5	69.5	68.5
Yak	69.5	68.5	67.5

Figure 2

- A random forest classification method was used to determine variable importance when classifying team win totals.
- %IncMSE shows how much the classification improves from node to node within the random forest when a given variable is used.
- The higher the %IncMSE the more important the predictor is to the classification.
- Important variables were then ran through a linear regression.
- Predictive model for wins was generated.
- Predictor variables in the regression were eliminated one by one based on insignificant p-values.
- Only R, H, TB, OBP, OPS, RA, IP, and SHO remained in the final, valid regression model.

## Results

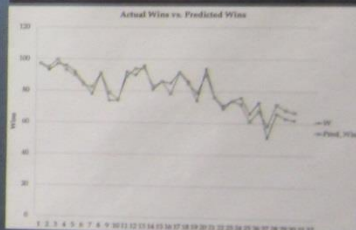


Figure 3

- The 2013 data was removed from the data set.
- The entire analysis was redone using the same methods in order to determine the effectiveness of the model.
- The resulting model had the same predictor variables as the original.
- A chi-square statistic was calculated to determine the strength of the model.
- Chi-Square statistics are used to determine significant differences between observed and predicted data.
- Chi-Square value of 5.4 and 29 degrees of freedom.
- accept the null hypothesis that there is no significant difference between actual and predicted win totals.
- Model does an extremely good job of predicting win totals.

After putting the model through...  
y = -281.34617  
- 987.7062(O

The calculated...  
RA 45.81616  
R 22.17066  
OPS 22.68347  
OBP 18.59176  
TB 15.68563  
OBP 13.85297  
IP 11.68505  
H 9.07950

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ing prices of different equity normal distribution. In his PhD pped modified Student t-losing prices of the Dow in the el for predicting closing prices and stock exchanges like the NASDAQ rmined that the standard Student of freedom often gave the best

uction

der to better predict closing prices at the financial marketplace. For a distribution of the wealth relative of e of the normal distribution. Dick hat the ends of the wealth relative normal distribution. Upon learning ion, a transformed student t icted future closing prices.

il and test his distribution against the g averages and degrees of freedom in distribution for predicting returns.

$$t(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{n}{2}\right)} \left(1 + \frac{x^2}{n-2}\right)^{-\frac{n+1}{2}}$$

Equation 2: Kuiper's Student T

ethods

Microsoft VBA and the first operation with relative returns, known as lambda, is equation below:

$$\lambda = \frac{\sum_{i=1}^n \lambda_i}{n}$$

# Using Different Student T-Distributions to Predict Closing Prices of Domestic Stocks

Tyler Higgins and Faculty Mentor: Dr. Ed Conjura  
The College of New Jersey Department of Mathematics and Statistics



The College of New Jersey

## Method Continued

The program then loops through a period count and calculates a mean ( $\mu$ ), using moving average, and a standard deviation ( $\sigma$ ). These calculations are crucial for predicting future closing prices, as well.

$$\mu = \sum_{i=1}^n \frac{\lambda_i}{n}$$

Equation 4: Moving Avg.

$$\sigma = \sqrt{\sum_{i=1}^n \frac{(\lambda_i - \mu)^2}{n-1}}$$

Equation 5: Standard Deviation

A histogram is created for each closing price, using a determined histogram radius. The count of each bin divided by the total number of data points used in creating the histogram returns the area of each bin. From here, the area of each bin is divided by the diameter of each histogram to return the histogram height. The height of each bin in the histogram is used to predict how many standard deviations away from the mean our predicted lambda will be. This is referred to as delta. The histogram height is compared to the values of each as delta. The histogram with varying degrees of freedom. The bisection method distribution with varying degrees of freedom. The bisection method is used to determine the closest density value to the height, and the corresponding x-value is used as delta. An illustration can be seen to the right.

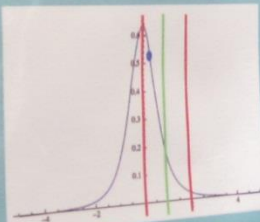
$$\text{Area} = \text{Number In Bin} / \text{Total In Histogram}$$

Equation 6: Histogram Area

$$\text{Height} = \text{Area} / \text{Width}$$

Equation 7: Histogram Height

Figure 2: Bisection Algorithm



After the predicted delta is computed using the best period count and best degrees of freedom, the predicted lambda is given using the following equation.

$$\lambda = \mu + \hat{\Delta} \sigma$$

Equation 8: Predicted Lambda

## Conclusion and Future Work

The plots chosen in the Results section show that this method is a pretty good predictor for the domestic stocks. It is important to note two specific results from the tables beneath the plots. The first is that the best lambda used in each case was the standard Student t-distribution, while the second is that the best degrees of freedom was consistently 2.

For the future, a more intensive analysis should be done on smaller samples of the data. For instance, a sample from 2008 would really test the accuracy of this method, as the stock prices were especially volatile at that time.

## References

- [1] Thorp, Edward. "The Distribution of Stock Price Changes." *Wilmott Magazine* 2002.
- [2] Thorp, Edward. "The Distribution of Stock Price Changes." *Wilmott Magazine* 2002.

## Acknowledgements

I would like to thank Dr. Ed Conjura for his valuable guidance throughout the course of this project.

I would also like to thank Elizabeth Swerney for her helpful and positive attitude while working with me on this project.

Finally, I would like to thank the Department of Mathematics for giving me exposure to an independent research project.

## Results



# Dining Customer Satisfaction at TCNJ

Tanairy Diaz, Professor Richard Thayer  
The College of New Jersey, Ewing NJ 08628



Correlations for Overall Food Scores

The CSRS Procedure

1. WMA Variables: 12  
2. Descriptions: 38 33 34 35 36

Variable	N	Mean	Std. Dev.	Sum	Minimum	Maximum	Label
1	471	1.5663	1.0252	735.9	1.0000	3.0000	1
2	470	1.9324	1.0814	905.9	1.0000	3.0000	2
3	466	1.9359	1.1547	902.1	1.0000	3.0000	3
4	466	1.8770	1.0780	874.2	1.0000	3.0000	4
5	462	2.0754	1.0817	958.3	1.0000	3.0000	5
6	457	2.0819	1.1707	959.9	1.0000	3.0000	6
7	457	2.0819	1.1707	959.9	1.0000	3.0000	7

Pearson Correlation Coefficients

Number of Observations: 471

	1	2	3	4	5	6	7
1	1.0000						
2	0.1200	1.0000					
3	0.1000	0.0800	1.0000				
4	0.0500	0.0500	0.0500	1.0000			
5	0.1500	0.1500	0.1500	0.1500	1.0000		
6	0.1000	0.1000	0.1000	0.1000	0.1000	1.0000	
7	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	1.0000

Correlations for Overall Service Scores

The CSRS Procedure

1. WMA Variables: 11  
2. Descriptions: 31 32 33 34 35

Variable	N	Mean	Std. Dev.	Sum	Minimum	Maximum	Label
8	466	1.9239	1.1547	894.4	1.0000	3.0000	8
9	462	1.9207	1.0816	889.5	1.0000	3.0000	9
10	457	1.9207	1.0816	889.5	1.0000	3.0000	10
11	457	1.9207	1.0816	889.5	1.0000	3.0000	11
12	457	1.9207	1.0816	889.5	1.0000	3.0000	12
13	457	1.9207	1.0816	889.5	1.0000	3.0000	13

Pearson Correlation Coefficients

Number of Observations: 466

	8	9	10	11	12	13
8	1.0000					
9	0.1000	1.0000				
10	0.1000	0.1000	1.0000			
11	0.1000	0.1000	0.1000	1.0000		
12	0.1000	0.1000	0.1000	0.1000	1.0000	
13	0.1000	0.1000	0.1000	0.1000	0.1000	1.0000

Correlations for Overall Dining Environment

The CSRS Procedure

1. WMA Variables: 11  
2. Descriptions: 31 32 33 34 35

Variable	N	Mean	Std. Dev.	Sum	Minimum	Maximum	Label
14	466	1.8271	1.0252	850.5	1.0000	3.0000	14
15	466	1.8271	1.0252	850.5	1.0000	3.0000	15
16	466	1.8271	1.0252	850.5	1.0000	3.0000	16
17	466	1.8271	1.0252	850.5	1.0000	3.0000	17
18	466	1.8271	1.0252	850.5	1.0000	3.0000	18
19	466	1.8271	1.0252	850.5	1.0000	3.0000	19

Pearson Correlation Coefficients

Number of Observations: 466

	14	15	16	17	18	19
14	1.0000					
15	0.1000	1.0000				
16	0.1000	0.1000	1.0000			
17	0.1000	0.1000	0.1000	1.0000		
18	0.1000	0.1000	0.1000	0.1000	1.0000	
19	0.1000	0.1000	0.1000	0.1000	0.1000	1.0000

## Analysis and Results

Frequency Tables Analysis:

Categories	Satisfied		Dissatisfied	
	Most Satisfied	Least Satisfied	Most Satisfied	Least Satisfied
Food	Taste	Freshness	Eye Appeal	Taste
Menu	Menu Choices	Healthy Menu Choices	Hearty Menu Choices	Menu Choices
Service	Friendliness of Staff	Hours of Operation	Hours of Operation	Speed of Service
Cleanliness	Serving areas & Eating areas	Layout of Facility	Eating Areas	Serving Areas
Dining Environment	Availability of Seating	Location	Availability of Seating	Location

Spearman Rank Correlation Analysis

Category & Subst	r	Category & Subst	r
Food -Taste	.78	Cleanliness -Serving area	.81
Menu -Menu Choice	.57	Dining Environment	.80
Service -Service	.776	Appearance	
Help of Staff			

## Conclusion

- Overall the dining services at The College of New Jersey is succeeding at keeping the student body satisfied: Percentages of responses in general categories with ratings of 3 or 2s (dissatisfied).
  - Overall Food: <20%
  - Overall Cleanliness: <13%
  - Overall Dining Environment: <5%
- There was a trend in the data in the Frequency Tables, which showed that subsets of the general categories most highly rated by the satisfied participants or vice versa were also the lowest rated by the dissatisfied participants in the overall food.
  - Taste was the subset most highly rated in the overall food by the category by the satisfied participants while it was the lowest rated by the dissatisfied participants.
  - Similar results were consistent with the other general categories, except for overall Dining Environment where both parties were dissatisfied with the availability of seating.
- Spearman Rank Correlation showed that these same subsets that appeared in the trend from the Frequency Tables were the most highly correlated to the satisfaction of their general categories.
  - Taste was the most highly correlated subset of the Overall Food category in terms of satisfaction with a correlation coefficient of 78%.

## Recommendations to Dining Services

- The data shows that the small percentage of the students who seem to be dissatisfied with certain aspects of the dining services are the same aspects that the majority of the students are satisfied with.
  - How can this be applied?

- Most highly correlated subsets are: Taste, Variety of Menu Choice, Helpfulness of Staff, Serving Areas and Appearance.
- When choosing the new establishments to be built in the student center these are the main subsets that should be focused on to increase overall satisfaction. In an effort to reach out to the small dissatisfied population perhaps better menu choices that are not in the dining hall would entice them to try seem to want something completely different. More available seating would definitely be a plus seeing as it was the only subset where both the satisfied and dissatisfied group were both not satisfied with.

building the customer order to gain aspects of the food, menu, but using a variety satisfied. Our ing for order to achieve by analyzing the its that could r choices. I analyze using SAS the subset for each, the customers.

CS

4/896

0

6

-67

-9

67

ff-40

First Year-177  
Sophomore-191  
Junior-124  
Senior-68  
Graduate-4

On Campus-488  
Off Campus-127  
Other-3

00

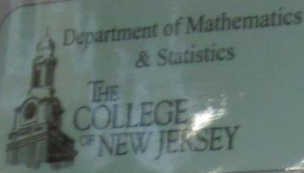
der-0

ntity-4

Table of 50 by 50

	1	2	3	4	5	Total
1	1	0	0	0	0	1
2	0	1	0	0	0	1
3	0	0	1	0	0	1
4	0	0	0	1	0	1
5	0	0	0	0	1	1
Total	1	1	1	1	1	5

55



# Using Time Series Analysis to Predict the Exchange Rate of the Japanese Yen to the US Dollar

Kelli Hyslop

## Abstract

An exchange rate is the amount of one currency that is required to purchase one unit of another currency. The actual exchange rate of any currency is determined by the forces of supply and demand for that currency in the world currency markets. As the demand for a currency on the foreign exchange market increases, the price of that currency rises, and an increase in interest rates, the strength of the exchange rate increases. Exchange rates affect our daily activities on a micro level, such as the price of imports and exports such as an increase in interest rates, the strength of the exchange rate increases. Exchange rates indirectly affect a country's Gross Domestic Product (GDP) and its balance of trade. From a macro perspective, exchange rates are also of interest to someone as small as a wholesaler and as large as an entire business. Traders who wish to invest or trade across global markets are also influenced by exchange rates. The purpose of this research is to find the best method of forecasting exchange rates in order to better predict future ones. If you are able to accurately predict future rates, you can take advantage of minimal information to improve the bottom line on investments. A variety of time series analysis methods will be applied in an attempt to find a good prediction of future exchange rates.

## Methods

Many different methods of forecasting have been executed over the years. Due to the difficulty that is forecasting exchange rates, no one method has been determined as superior, although, a small group of them are among the most popular. These methods generally fall in one of two categories: Fundamental or Technical. Initially, a mixed approach, utilizing time series analysis and a chosen fundamental method, was desired to be used for this analysis. After much research, a decision was made to only involve the technical method for forecasting the Yen per US Dollar exchange rate. The technical method performed was time series analysis; more specifically, triple exponential smoothing.

$$\hat{Y}_{t+m} = (\hat{\alpha}_t + m\hat{\beta}_t + \hat{\gamma}_t)_{t-1+m}$$

where  $m > 0$  is a chosen # of days to forecast out,

$$\hat{\alpha}_t = \alpha \cdot \frac{2}{L+1} + (1-\alpha)(\hat{\alpha}_{t-1} + \hat{\beta}_{t-1})$$

$$\hat{\beta}_t = \gamma(\hat{\alpha}_t - \hat{\alpha}_{t-1}) + (1-\gamma)\hat{\beta}_{t-1}, \quad \&$$

$$\hat{\gamma}_t = \beta \cdot \frac{2}{L+1} + (1-\beta)\hat{\gamma}_{t-1}$$

The goal here was to find the best alpha, beta, and gamma that will produce the smallest Mean Absolute Deviation, MAD. MAD =  $\frac{\sum |e_t|}{n}$ . Next, the use of the random walk method was implemented. This was to determine if there was a trend in the data, or simply random noise. With the exchange rate data, the differences were calculated between the current day and the previous day. From here, the maximum and minimum values were found, and a random number generator was performed between these two values. These would become the error values,  $\epsilon$ , and the random walk would be calculated by  $\hat{Y}_t = \hat{Y}_{t-1} + \epsilon$ .

## Results

The data used for this section of the analysis was provided by a financial professional to Dr. Conjura.

The random walk method was performed to the Japanese YEN/US Dollar exchange rate data. The list of daily exchange rates was chosen for a certain period of time. After the random walk was performed, its MAD error was calculated. The random walk data and the original exchange rate data were then plotted on top of one another.



As seen in figure 1, it is clear that the original exchange rates are less volatile than the random walk. Based on the analysis of the errors and the graphs, it is clear that the exchange rates have some sort of trend in the data and not just white noise.

Triple Exponential Smoothing was used to forecast the exchange rate of the Japanese Yen per US Dollar. The number of days to forecast out was chosen to be 30. With the first exchange rate, the first prediction is calculated for the 30th day, until the last data point. The best parameters were found to be  $\alpha = 0$ ,  $\beta = 1$ , and  $\gamma = 0$ . Then, the predictions were compared to the corresponding original value, using MAD. This method proves to be a good predictor of future values.

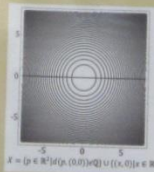


Figure 2



**Abstract**

A proper dense connected subgroup of the plane is constructed. To this end we introduce ordinal numbers, transfinite recursion, and two results about subsets and subgroups of  $\mathbb{R}^2$ .



$$X = \{p \in \mathbb{R}^2 \mid (p, (0,0)) \in \mathbb{Q} \cup \{(x,0) \mid x \in \mathbb{R}\}\}$$

	Three out of Four			
	Proper	Dense	Connected	Subgroup
$\mathbb{R}^2$	Red	Green	Green	Green
Lines (through origin)	Green	Red	Green	Green
$\mathbb{Q}^2$	Green	Green	Red	Green
$X$	Green	Green	Green	Red

**Lemma 1**

If  $A$  is a subset of the plane that intersects every uncountable closed subset then it is dense and connected.

**Proof Sketch**

- Density follows from the fact that in  $\mathbb{R}^2$  every nonempty open set contains an uncountable closed subset
- If  $U$  and  $V$  are disjoint open sets then  $\mathbb{R}^2 - (U \cup V)$  is an uncountable closed set

**Lemma 2**

If  $A \subseteq \mathbb{R}^2$  and  $B \subseteq \mathbb{R}^2$  such that  $A \cap B = \emptyset$  and  $S_{(A,B)} = \{x \in \mathbb{R}^2 \mid (A,x) \cap B \neq \emptyset\}$  then  $|S_{(A,B)}| \leq \max\{|A|, |B|, \aleph_0\}$ .

**Proof Sketch**

- Define  $\varphi: A \times B \times \mathbb{Z} \rightarrow \mathbb{R}^2$  by  $\varphi(a, b, n) = \frac{1}{n}(b - a)$
- If  $x \in S_{(A,B)}$  there are  $a \in A, b \in B, n \in \mathbb{Z}$  such that  $b = a + nx$  with  $n \neq 0$
- Thus  $x = \frac{1}{n}(b - a)$  and  $S_{(A,B)} \subseteq \varphi[A \times B \times \mathbb{Z}]$  which has cardinality  $\max\{|A|, |B|, \aleph_0\}$

**Ordinal Numbers**

Ordinal numbers are an extension of the natural numbers, different from integers and cardinal numbers, and correspond to the order type of well-ordered sets.

$$0, 1, 2, \dots, 1024, \dots, \omega, \omega+1, \dots, \omega+\omega, 2\omega+1, \dots, \Omega, \dots$$

**A Proper Dense Connected Subgroup of  $\mathbb{R}^2$**

**Recursion**

- Define the  $0^{\text{th}}$  term of a sequence
- Define how, given the  $n^{\text{th}}$  term, one would calculate the  $(n+1)^{\text{th}}$  term

**Recursion v. Transfinite Recursion**

**Transfinite Recursion**

- Define the  $0^{\text{th}}$  term of a sequence
- Successor: Define how, given the  $n^{\text{th}}$  term would calculate the  $(n+1)^{\text{th}}$  term
- Limit: Define how, given all terms preceding the  $\alpha^{\text{th}}$  term, one would calculate the  $\alpha^{\text{th}}$  term

**Desired Properties**

- Each  $H_\alpha$  is countable
- $H_\alpha \subseteq H_\beta$  for  $\alpha < \beta$
- $H_\alpha \cap C_\beta \neq \emptyset$  for  $C_\beta \in \mathfrak{C}$ , the set of uncountable closed sets, and each  $\alpha, \beta \in \Omega$  with  $\alpha < \beta$
- For each  $\alpha \in \Omega$  there is a set  $E_\alpha$  containing elements  $y_\beta$  for all  $\beta \leq \alpha$  such that  $E_\alpha \cap H_\gamma = \emptyset$  for all  $\gamma < \alpha$

**Initial**

- $\mathfrak{C}$  can be well-ordered:  $C_0, C_1, \dots, C_{\omega+1}, \dots$
- Fix  $H_0 = \emptyset$  and let  $y_0 \in C_0$
- Let  $x_0$  be any point in  $C_0$  so that  $(x_0)$  misses  $\{y_0\} = E_0$
- Let  $H_1 = (x_0)$  and choose a  $y_1 \in C_1 - H_1$

**Successor**

- In general, if  $H_{\gamma-1}$  and  $y_{\gamma-1} \in C_{\gamma-1}$  constructed and chosen then:
- Choose a  $x_{\gamma-1} \in C_{\gamma-1} - H_{\gamma-1}$  so that  $(x_{\gamma-1})$  misses  $\{y_0, y_1, y_2, \dots, y_{\gamma-1}\} = E_{\gamma-1}$
  - Let  $H_\gamma = (H_{\gamma-1}; x_{\gamma-1})$
  - Choose a  $y_\gamma \in C_\gamma - H_\gamma$

**Limit**

If a limit ordinal then  $H_\beta$  has been defined for all  $\beta < \alpha$ . Let  $H_\alpha = \bigcup \{H_\beta \mid \beta < \alpha\}$ . countable union of countable sets so there is certainly a  $y_\alpha \in C_\alpha - H_\alpha$

**Proper Dense Connected Subgroup**

Let  $H = \bigcup \{H_\alpha \mid \alpha \in \Omega\}$  and  $E = \bigcup \{E_\alpha \mid \alpha \in \Omega\}$ .  $H$  is a union of proper subgroups intersects every element of  $\mathfrak{C}$  and entirely misses  $E \neq \emptyset$ .

Thus,  $H$  is a proper dense connected subgroup of the plane.

**References**

- A. Clifford. Infinitely many pairwise disjoint connected dense subgroups of the plane. *Journal of Mathematics* 31
- E.S. Thomas. Connected subgroups of Lie groups. *Illinois Journal of Mathematics* 31



# Continuous Fair Division Scheme

Emily Gutterson, Advisor: Andrew Cliff

## TCNJ

### Abstract

We often deal with the problem of having to divide a set of goods between a group of people. These problems are formally known as fair division problems.

We will explore a few different procedures that solve fair division problems, called fair division schemes. Divisions are considered more effective when they satisfy more properties. We will look at two different continuous fair division schemes, the Divide and Choose Method and the Last Diminisher Method. We will then explore the properties that they satisfy or do not satisfy. When faced with a fair division problem, deciding between which scheme to apply depends on which properties are more crucial.

### Continuous versus Discrete

A **Continuous set** is a set that can be easily divided in an infinite amount of ways.

Example: Pizza, Cake, Land, Money

A **Discrete set** is a finite set that cannot easily be divided.

Example: An estate consisting of a car, house & a boat.

### Valuations

Each player  $i$  has a personal valuation function.

$$V_i: P(X) \rightarrow [0,1]$$

If  $X_1, X_2, \dots, X_n$  is a partition of  $X$ , then  $\sum_{k=1}^n V_i(X_k) = 1$

### Divide & Choose Method

One player cuts; the other player chooses.

The player making the cut divides the set into two parts, between which he/she is indifferent.  $V_A(X_1) = V_A(X_2) = 0.5$

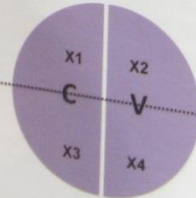
### Case Study

Assume 2 players want to divide a cake fairly.

Player A,  $P_A$ , likes vanilla three times as much as chocolate.

Player B,  $P_B$ , likes vanilla and chocolate equally.

Player A is the divider.  
Player A divides the cake along the vertical line as shown.  
Player B chooses the piece which he/she values more.



	Slice 1 = $X_1 + X_2$	Slice 2 = $X_3 + X_4$
$V_A$	0.5	0.5
$V_B$	0.5	0.5

### Case 2

- Player B is the divider.
- Player B divides the cake along the vertical line, into two indifferent pieces,  $X_1$  and  $X_2$ .
- Player A chooses the piece which he/she values more.

	Slice 1 = $X_1$	Slice 2 = $X_2$
$V_A$	0.25	0.75
$V_B$	0.5	0.5



### Case 3

- This situation will never happen unless there is some outside influence.
- $X_1$  is 50% of the cake.
- $X_2$  is 10% of the cake.
- $X_3$  is 40% of the cake.

Neither player is indifferent between the two slices.

	Slice 1 = $X_1 + X_2$	Slice 2 = $X_3$
$V_A$	0.4	0.6
$V_B$	0.6	0.4



### Fair Division Properties

#### PROPORTIONAL

$$V_i(X_i) \geq \frac{1}{n} \text{ for all } i$$

#### ENVY-FREE

$$V_i(X_i) \geq V_i(X_j) \text{ for all } i \text{ and } j$$

#### EQUITABLE

$$V_i(X_i) = V_j(X_j) \text{ for all } i \text{ and } j$$

#### PARETO-OPTIMAL

There is no other division that is better for one player and at least as good for other players.

### Properties of the Divide & Choose Method

PROPORTIONAL (ALWAYS)  
Case 1 & Case 2

EQUITABLE (SOMETIMES)  
Case 1 & Case 3 (Not Case 2)

ENVY-FREE (ALWAYS)  
Case 1 & Case 2

PARETO-OPTIMAL (SOMETIMES)  
Case 2

Theorem 1: If a fair division method is PROPORTIONAL, EQUITABLE, and PARETO-OPTIMAL, then it is ENVY-FREE.

5

### Conclusion

After putting the important variables used in the model through one last random forest classification, we as the dependent variable, it was clear that the model was not as good as we thought it was. This was due to the way the model was built. In terms of accuracy, the model was not as good as we thought it was. In terms of prediction, the model was not as good as we thought it was. In terms of interpretation, the model was not as good as we thought it was.



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TCNJ

# Strategizing the Adjusted Winner Procedure in a Company

Christopher Falk  
Dr. Andrew Clifford

**Abstract**  
For division problems, it is necessary to carefully choose a procedure to use in the situation. In some cases, the Adjusted Winner Procedure (AW) is superior. There can be a tie for best of goods, and the goods can be continuous, discrete, or mixed. An allocation rule of the AW procedure can be presented in 3 steps: 1. Allocation, 2. Equitability adjustment, and 3. Point assignment.

**Example**  
Tech Co. is a mixed and contains both... of primary HQ, appointment of... (employee layoffs)

	Tech Co
Chairman	20
CEO	25
Immunity to EE layoffs	10
Totals	55

**2. Transfer stage**  
So far, Sancorp won 40 points. Tech Co won 35. Transfer the tied goods to Tech Co. Tech Co becomes the *Temporary Winner*. Analogously, Sancorp becomes the *Temporary Loser*.

**Analysis by Point Ratios**  
• Definition – Tech Co's *Point Ratio* for a good is Tech Co's valuation of the good divided by Sancorp's valuation.  
• If Tech Co's point ratio for a good is X, we say, "Tech Co values the good X-times more than Sancorp."  
• Transfer goods to Sancorp in ascending order of Tech Co's point ratios [lowest PR's → highest PR's]

	Sancorp	Tech Co
Company name	40	
Primary location		25
Chairman		10
CEO	15	30
Immunity to EE layoffs		
Totals	55	65

**3. ★ Equitability adjustment ★**  
• This step is crucial. The equitability adjustment is unique to the AW procedure, and guarantees equitability with absolute certainty.  
• Equitability means that each player values their allocation exactly equal to the other.  
• This is often difficult to achieve with fair division schemes.

**Solving for p**  
• Definition – p is the percentage of the CEO good Sancorp needs to receive to achieve an equitable allocation.  
$$55 + 30p = 35 + 30(1-p)$$
  
$$\rightarrow p = 1/6$$

	Sancorp	Tech Co
Company name	40	
Primary location		25
Chairman		10
CEO	30(1/6) = 5	25
Immunity to EE layoffs	15	30
Totals	60	65

## Other Con

**1. Manipulation**  
• Highly difficult to unf...  
• Strong condition must...  
• exact point valuation...  
• Allocation = Envy-free...  
• Effectively, you win me

**2. Honesty allo**  
• The optimal outcome...  
1. Announce...  
2. Have polar...

Consider the following ex

	Sancorp	Tech Co
Company name	40	
Primary location		25
Chairman		10
CEO	15	30
Immunity to EE layoffs		
Totals	55	65

	Sancorp	Tech Co
Co. name	40	10
Location	5	35
Chairman	5	25
CEO	40	5
EE layoffs	10	25
Totals	100	100

$80 + 10p = 60 + 25(1-p)$   
 $\rightarrow p = .143$



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# Forecasting the US Dollar EU Euro Exchange Rate using Technical Approaches

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## Abstract

An exchange rate is a conversion rate used to determine the amount of one country's currency needed to obtain one unit of another country's currency. Exchange rates affect our daily activities on a state and macro scale. In the macro perspective, exchange rates indirectly affect a country's Gross Domestic Product (GDP). On a micro scale, an exchange rate affects spending activity in a foreign market because anyone (business/individual) who is selling something or providing a service needs their country's currency. Therefore, being able to forecast future exchange rates is a skill desired by many.

Not only are these predictions of use to individuals as well as businesses who wish to improve the bottom line of investments, exchange rates can also influence market activity in an economy. With the recent financial crisis of 2008, monetary/fiscal policies to keep their economy stabilized.

However, predicting exchange rates is near impossible and professionals use technical approaches used to forecast exchange rates to improve

## Method

Many different methods of forecasting have been associated over the years. These methods generally fall in one of two categories: fundamental or technical. Technical approaches include purchasing power parity and the random walk method. However, the technical approach of time series analysis was selected to execute this research.

Time series analysis uses data recorded over time to discover if a pattern exists among the given data. Four time series analysis uses the pattern to predict future values. In order to capture a trend, double exponential smoothing was used to make predictions.

$$\begin{aligned}
 (1) \quad & \hat{h}_t = \alpha y_t + (1 - \alpha)(\hat{h}_{t-1} + \hat{h}_{t-2}) \\
 (2) \quad & \hat{h}_t = \beta(\hat{h}_{t-1} - \hat{h}_{t-2}) + (1 - \beta)\hat{h}_{t-1} \\
 (3) \quad & \hat{h}_{t+1} = \hat{h}_t + m\hat{h}_t \\
 \end{aligned}$$

Where  $m = 1/\theta$  (number of days predicted),  $\hat{h}_t$  is the unadjusted forecast,  $\hat{h}_t$  is the adjusted trend, and  $\hat{h}_{t+1}$  is the prediction.

The percentage values that produced the best results ("fit") were used for the rest of the project. To measure the "fit" of this method, and comparative methods, the mean absolute deviation was calculated.

$$MAD = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{n}$$

Where  $n$  is the number of values being calculated.

To find the best method, predicted exchange rate values were obtained using a random walk technique and rates predicted by professionals.

The random walk was used to determine if a trend was present in the exchange rate or other factors. To produce the random values, the next day's exchange rate was determined from the current day's rate. The maximum and minimum values of data differences were used to the upper and lower bounds for the random number generator. Then, these random numbers became an array that the next day's value is added to the prior rate to predict tomorrow's value.

$$\hat{P}_t = P_{t-1} + \hat{h}_t$$

## Results



Figure 1 (above) displays the best 30-day "fit" forecasted USD/EUR values using double exponential smoothing where  $\alpha = 0.6$ ,  $\beta = 0.1$ , and  $MAD = 0.00719365$ .

The data points used throughout this project were obtained from my faculty advisor, Dr. Edward Conjura, who worked in the corporate world. The actual exchange rates were ultimately provided by financial professionals from Dr. Conjura's network. The specific data points chosen for this project, fall between the dates April 1<sup>st</sup>, 2009 and May 1<sup>st</sup>, 2010. During this time, the US stock market was rebounding from a devastatingly decreasing trend due to the financial crisis. This same time period is used throughout the project to maintain consistency.

Figure 2 (below) illustrates the random walk method where  $MAD = 0.015547$ , \*assumed  $P_{t+1} - P_t$  is uniformly distributed



Figure 4: Smoothing (fit)  $MAD = 0.0081$

## Analysis

The technical smoothing

$\pi$

## Mathematics Education Independent Study Fall 2013-2014

### > Literature Review

#### - Students' Understanding of Right Triangle Trigonometry

- > Emphasize the conceptual knowledge rather than solely rote procedures and mnemonic devices

#### - The Use of Multiple Representations

- > Differentiate instruction
- > Motivate students to analyze mathematics in various ways

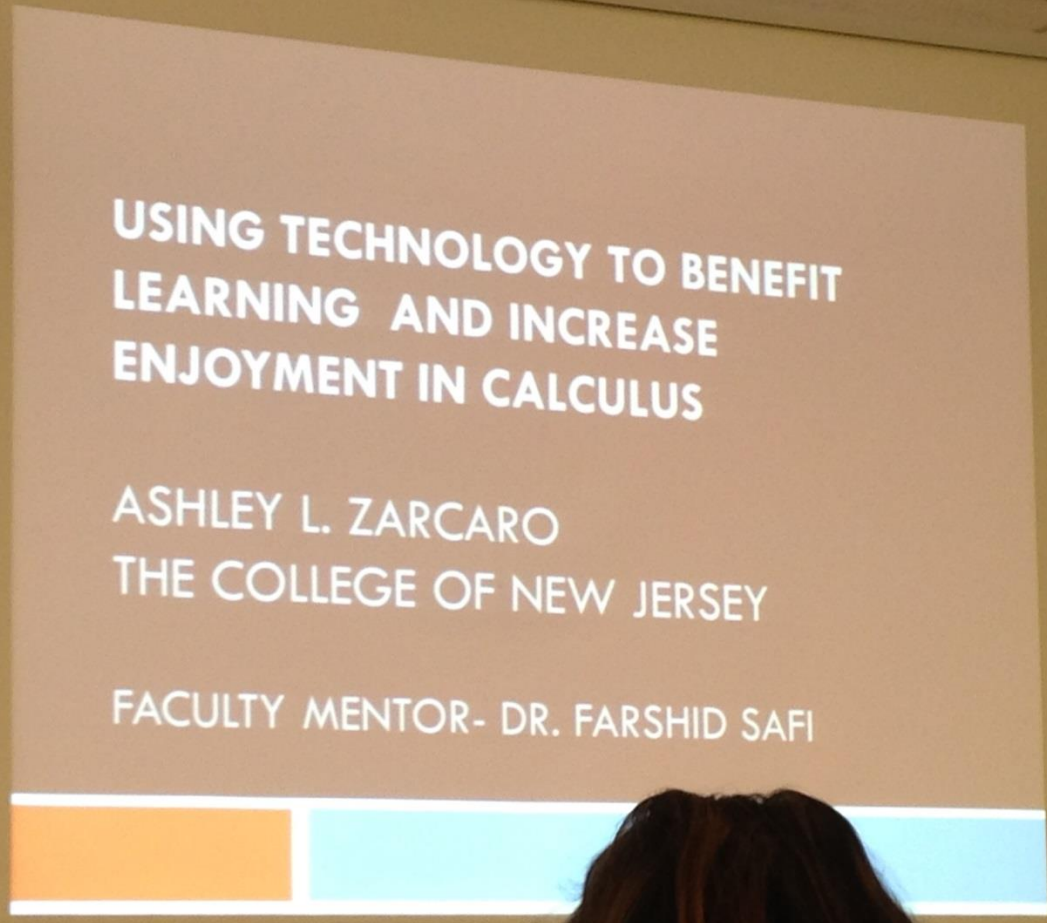
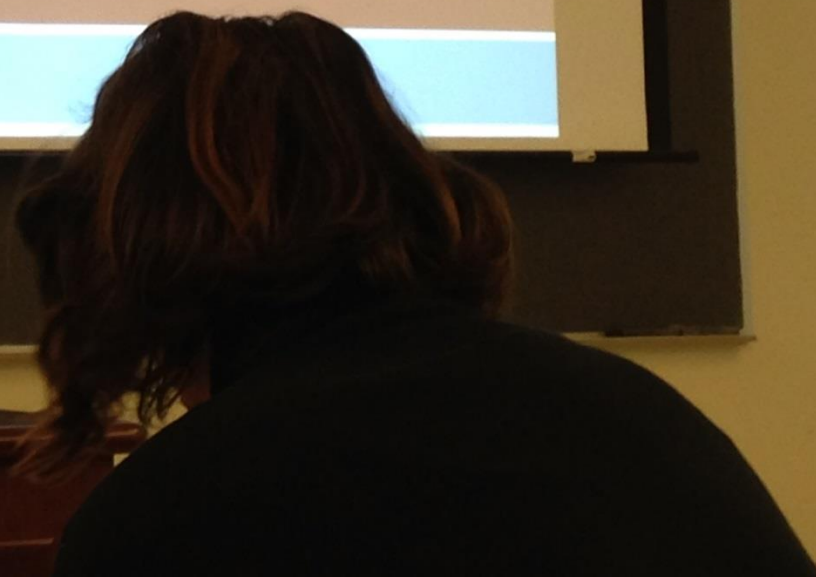
#### Framework: TPACK (Technology, Pedagogy, and Content Knowledge)

- > Embrace change and learn to use technology appropriately and effectively

**USING TECHNOLOGY TO BENEFIT  
LEARNING AND INCREASE  
ENJOYMENT IN CALCULUS**

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THE COLLEGE OF NEW JERSEY

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A large projection screen displays a website for Reynolds Middle School. The page features a header with the school's name and a logo of an eagle with wings spread, perched on a globe. Below the logo is the text "REYNOLDS RAIDERS" in a stylized font with a red, white, and blue striped pattern underneath. The website content includes sections for "LOCATION", "POPULATION", "WORKSHEET", and "TECHNOLOGY".

Reynolds Middle School, Hamilton, New

**LOCATION**  
Reynolds Middle School is located in a very large and diverse town. The school, and 7 high schools.

**POPULATION**  
Reynolds consists of 1,000 students. There is a 17:1 student to teacher ratio.

**WORKSHEET**  
There are many extra-curricular activities available for every student. Parents are encouraged to be involved. Parents are encouraged to be involved.

**TECHNOLOGY**  
There is an impressive amount of technology at this middle school. Smart Board, interactive whiteboards, laptops are available for students' use if the teacher is using a computer. Graphing calculators available for every student.

http://www.weebly.com/teaching-and-great.html

Some examples from my student teaching

- Algebra II: The class as a whole would determine how to find formulas for finding terms and sums of arithmetic and geometric sequences and series. The Honors students would always ask where these formulas came from, so this satisfied their curiosity.
- Math Analysis: Verifying trigonometric identities requires a lot of proof based skills. Students struggled with these skills, so I tried to give them some pointers when we worked on these problems.

### VERIFYING TRIGONOMETRIC IDENTITIES

- GOAL: Start with the left hand side, and use identities and algebra to transform it to the right hand side.
- If there are squares use a Pythagorean identities.
- Change everything to sine and cosine.
- Separate a single fraction into two separate fraction with a common denominator, their reciprocals.
- If there are two fractions being added or subtracted, use the lowest method to add them simply.
- Don't be afraid to go backwards to try something else if what you're doing is not working.

Scribd



$$\text{Vol (cylinder)} = \frac{4}{3} \pi r^3$$
$$\text{Area (sphere)} = 4 \pi r^2$$
$$\text{Vol (Cylinder)} = \pi r^2 h$$







**REFLECTION**

**Jerry**

- Graphic organizers really helped recall information.
- Showed some improvement in his communication of his mathematical reasoning.
- Discussions did not help as much since he was not always focused.
- Most likely will not continue to improve.

**Melissa**

- Discussions helped improve her confidence.
- Improved more than Jerry in her communication of her mathematical reasoning.
- She really like the graphic organizers, helped her study.
- May continue to improve.

**RECOMMENDATIONS**

- Must set a high expectation of recall at the beginning of the year.
- Teachers must maintain this high level of control and provide mathematical language.
- Maintain a classroom environment that encourages discussion.
- Graphic organizers helped improve the organization of notes.





Assessment 1

R's Assessment 1:

- Did not answer any questions
- When asked after class, he replied "I didn't understand what to do"

Law of Sines/Cosines Word Problems

1. A post is supported by two wires (one on each side going in opposite directions) creating an angle of  $90^\circ$  between the wires. The ends of the wires are 12m apart on the ground with one wire forming an angle of  $40^\circ$  with the ground. Find the lengths of the wires.
2. Two ships are sailing from Halifax. The Nina is sailing due east and the Pinta is sailing  $45^\circ$  south of east. After an hour, the Nina has travelled 115km and the Pinta has travelled 28km. How far apart are the two ships?
3. 3 friends are camping in the woods. Bert, Ernie and Elmo. They each have their own tent and the tents are set up in a triangle. Bert and Ernie are 10m apart. The angle formed at Bert is  $30^\circ$ . The angle formed at Elmo is  $105^\circ$ . How far apart are Ernie and Elmo?
4. Two scuba divers are 20m apart below the surface of the water. They both spot a shark that is below them. The angle of depression from diver 1 to the shark is  $30^\circ$  and the angle of depression from diver 2 to the shark is  $45^\circ$ . How far is the shark from the divers?
5. To estimate the length of a river, a surveyor walks 100m from point A to point B. He then turns and walks 150m from point B to point C. The angle at point B is  $120^\circ$ . Approximately how long is the river?
6. Two observers are standing on shore looking at a boat in the water. The angle of depression from the observer to the boat is  $30^\circ$  and the angle of depression from the other observer to the boat is  $45^\circ$ . The distance between the two observers is 50m. Find the distance from each observer to the boat.
7. Jack and Jill both start at point A. They walk in different directions. Jack walks  $100^\circ$  to reach point B. After 45 minutes Jill reaches point C. How far apart are they?



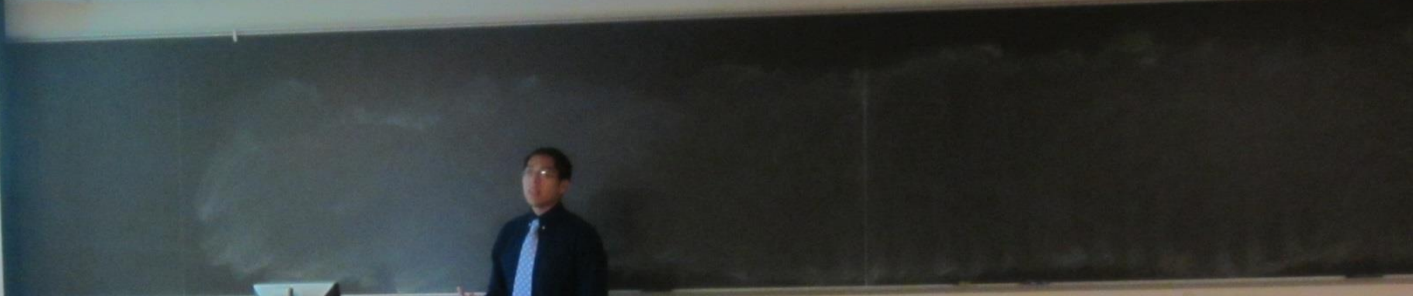
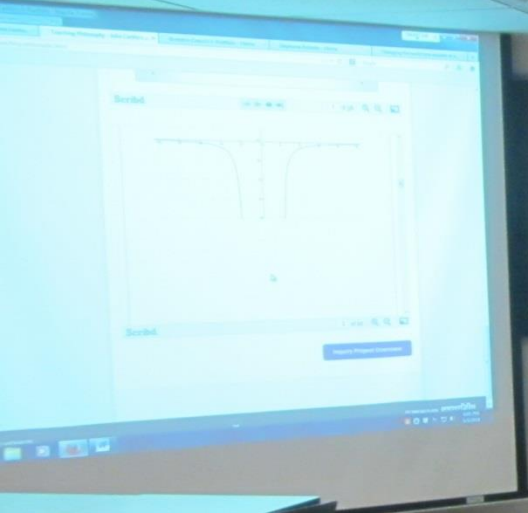
$$V_d(\text{sphere}) = \frac{4}{3} \pi r^3$$

$$A_{\text{area}}(\text{sphere}) = 4 \pi r^2$$

$$V_d(\text{Cylinder}) = \pi r^2 h$$









Learning Goals

- Understand the importance and benefits of visualization
- Comprehension of a problem statement
- First strategy: 1- Display concepts artistically, 2- Mathematical writing to display conceptual understanding
- Second strategy: Complete correctness with or without a picture given to them

Throughout this lesson, having both strategies... understand being shown how the students should draw a picture... statement to make a claim for... problem... Then there, the goal is to draw from the experience of visualization in a geometry problem... I want the students to realize that they first step is to draw a picture, which requires that they... the student's own perspective to solve a problem... I don't think I can teach this... understanding... having students will have the benefits of using visual aids in geometry for some problems.

Through my activities... have been able to help my students understand... visualization... visualization of the application... strategy will help them to solve the... Then, the goal is to draw from the experience of visualization in a geometry problem... I want the students to realize that they first step is to draw a picture, which requires that they... the student's own perspective to solve a problem... I don't think I can teach this... understanding... having students will have the benefits of using visual aids in geometry for some problems.

There are assessment for the first strategy... to draw... I don't think I can teach this... understanding... having students will have the benefits of using visual aids in geometry for some problems.

The second goal here for the students with the assessment is to draw... to draw... I don't think I can teach this... understanding... having students will have the benefits of using visual aids in geometry for some problems.

Peer-to-peer assessment for the second strategy... the goal is to provide... to draw... I don't think I can teach this... understanding... having students will have the benefits of using visual aids in geometry for some problems.

Vol (sphere) =  $\frac{4}{3}\pi r^3$   
Area (sphere) =  $4\pi r^2$   
Vol (Cylinder) =  $\pi r^2 h$





















AVANTAGE









