

Program Cover Document --- MAT 111 Number theory and Systems for Middle School Teachers

I. Learning Goals

Currently, this course is not a part of any program offered by the Department of Mathematics & Statistics. It is one of a series of courses offered to meet the needs of Elementary Education majors who wish to obtain the New Jersey middle school subject matter endorsement in mathematics. This requires 15 credits in mathematics and passing the Middle School Mathematics Praxis exam. Many students who have taken MAT 105 and/or 106, courses required for elementary education majors, wish to continue their study of mathematics in order to obtain the endorsement and teach mathematics in grades 6-8.

Teaching numbers and operations effectively at the middle grades requires that teachers have a deep and profound understanding of the number theory that is part of the K-12 curriculum. The main goal of this course is to enable middle grades preservice teachers to develop an advanced viewpoint of the numbers and operations that they will teach.

Each course in the series of courses that are offered to middle school mathematics teachers is focused on a particular content strand from the New Jersey Core Curriculum Content Standards (NJCCCS). The other courses are MAT 112, Data Analysis & Probability for Middle School Teachers, MAT 114, Patterns, Functions & Algebra for Middle School Teachers, MAT 115, Geometry for Middle School Teachers, MAT 117, Discrete Math for Middle School Teachers, and MAT 118, Concepts of Calculus for Middle School Teachers.

II. Student Assessment

Assessment in this course is similar to the assessment done in the prerequisites, MAT 105 and 106. Preservice teachers can demonstrate their content knowledge through a variety of assessment methods that require them to solve problems, explain their reasoning, and use mathematical representations.

III. Learning Activities

Middle school mathematics curricula are used as a basis for in-depth study of the mathematics content. Many class sessions are devoted to discussions of the mathematics content and the development of the mathematical concepts and skills necessary in the mathematical education of middle-school grades. Other class sessions consist of “hands-on” experiences in order for the preservice teachers to learn how to use concrete manipulative materials, pictorial models, and technology in the development of the mathematical concepts for middle-school grades. Students are expected to complete readings, and prepare projects and/or assignments that are appropriate for the course.

Using middle school mathematics curricula as motivation for in-depth study of the mathematical concepts connects the course content to the middle school classroom. Preservice teachers appreciate that what they are learning will be relevant to their teaching. Teachers must experience standards-based teaching and learning in order to understand how to implement it. A variety of strategies and methods of instruction should be used to model effective teaching of mathematics.

Departmental Course Syllabus – MAT 111 Number Theory and Systems for Middle School Teachers

I. Basic Information

- A. Purpose statement: Teaching numbers and operations effectively at the middle grades requires that teachers have a deep and profound understanding of the number theory that is part of the K-12 curriculum. The main goal of this course is to enable middle grades preservice teachers to develop an advanced viewpoint of the numbers and operations that they will teach. The course is geared toward elementary education majors who wish to get the New Jersey subject matter endorsement in mathematics to teach in grades 6-8.
- B. Course description: This course gives the middle school mathematics teacher a deeper understanding of numbers and operations. Physical materials, models, technology, and middle school curricula will be used to explore these topics. Exploration of ways to engage middle school students meaningfully in experiences that lay a strong foundation for more formal work in number theory will be integrated with the mathematics content.
- C. Course prerequisites: MAT 105 or 106 or by permission of Math Education coordinator

II. Learning Goals

- A. Content goals: Students will engage in a thorough development of the properties of the number systems, including integers, rational and real numbers, and number theory. Through this process, they will develop understanding of the processes and algorithms found in middle school mathematics and discover purposes beneath the symbols and techniques.
Many students will enter the course with the misconception that mathematics is all about computation and following procedures. Through careful consideration of many types of problems found in NSF-reform curricula, such as *Connected Math*, and alternative algorithms, students will come to understand that mathematics is much more than computation, and that there is often more than one approach to solving any problem.
Another common misconception is that one can prove that a mathematical statement is true by example. Through the study of number theory, students will learn that one needs to provide a logical argument to prove the truth of a mathematical statement.
- B. Performance goals: By the completion of the course, the successful student will be able to demonstrate all of the following:
 - Deeper understanding of the concepts, processes and algorithms, and the purposes beneath them, found in the middle school mathematics curriculum, with a focus on number systems and number theory.
 - Improved ability to communicate mathematical ideas appropriately using the language of mathematics.
 - Improved ability to reason mathematically and begin developing mathematical proofs.
 - Ability to approach problems involving number systems and number theory from multiple perspectives.
 - Knowledge of the relationship of mathematics to other subjects, its applications in society, and relationships within mathematics itself, in

particular, the relationship between all of the number systems included in the syllabus.

- Increased understanding of how children learn number concepts, the developmental and psychological foundations of how one teaches number concepts, and national and state standards relating to number and operations.
- Knowledge of the use of concrete manipulative materials, technology, and pictorial representations necessary in the development of number concepts and skills.
- Ability to construct strategies, which are supported by current research, for teaching number concepts to students of differing abilities.
- Increased ability to implement the five (NCTM) Process Standards: problem solving, reasoning and proof, communication, connections, and representation.

III. Student Assessment

- A. Assessment plan: Students will be assessed and receive regular feedback on their work through the assignment of homework, written and oral communication, group and/or individual projects, in-class examinations, and a final exam.
- B. Rationale: Homework assignments will provide students with opportunities to attempt lengthier, more challenging problems than is possible on an examination as well as offering students practice at exam-style problems. Written and oral communication will be used to directly assess some of the performance goals listed above. Group and individual projects provide students with the opportunity to explore a concept more deeply, and at the same time, assess many of the performance goals listed above. Examinations, which normally preclude both the use of books and the practice of group discussion, enable the professor to assess the knowledge an individual student has readily available.
- C. Methods and criteria: The final grade will be based on the following:
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| 30% | 3 quests (short tests) |
| 20% | group project |
| 20% | homework/class participation |
| 30% | final exam |

Quests and the final exam will cover correct use of mathematical terminology and mathematical concepts studied.

The project will allow students to explore a concept more deeply with connection to pedagogical issues. This will be due at the middle of the term.

Homework will be assigned regularly to reinforce the advanced mathematical concepts learned in class. A great deal of time in class will be spent on problem-solving activities from NSF-reform curricula, such as *Connected Math*, and class discussion of advanced concepts and their relationship to these problems.

Problem-solving rubric scale (used to describe performance on homework problems and quest problems):

- 4 Student demonstrates complete understanding of task or problem. Appropriate solution strategies were clearly written and correctly implemented to solve the problem. All work is mathematically correct and neat. A clear explanation and/or justification is provided if necessary.

- 3 Student demonstrates understanding of task or problem. Appropriate solution strategies were selected, but minor procedural errors were made while implementing the strategy, or the work was not complete. An explanation and/or justification is provided if necessary.

Or although work is mathematically correct, it is difficult to determine if the student understands the problem or task because there is unclear, little, or no explanation provided.
- 2 Student demonstrates at least partial understanding of task or problem, but there are significant errors in thinking. Work is at least partially correct.
- 1 Very little, if any, understanding of the task or problem is evident. Work is attempted, but is incorrect.

Written Work Rubric scale (used to describe performance on explanations or other written homework or quest questions)

- 4 Appropriate mathematical vocabulary was used. The explanation was specific and concisely identified the point(s) raised for discussion. The explanation was entirely correct.
- 3 There is a minor error in the use of vocabulary or in articulating the point raised for discussion. Otherwise the explanation is entirely correct and specifically and concisely identified the point(s) raised for discussion.
- 2 The point(s) raised for discussion was identified, however the writing was evasive and/or vague. Terminology was avoided or inappropriately used.
- 1 Inappropriate writing or no attempt.

IV. **Learning activities**

- A. Summary of learning activities: Middle school mathematics curricula will be used as a basis for in-depth study of the mathematics content. Many class sessions will be devoted to discussions of the mathematics content and the development of the mathematical concepts and skills necessary in the mathematical education of middle-school grades. Other class sessions will consist of “hands-on” experiences in order for the students to learn how to use concrete manipulative materials, pictorial models, and technology in the development of the mathematical concepts for middle-school grades. Students will be expected to complete readings, and prepare projects and/or assignments that are appropriate for the course.
- B. Calendar or outline: The following is a suggested guide to the organization of course topics.
 - What is a mathematical proof? Patterns and other problem solving strategies.
 - Divisibility
 - Primes, factorization, and the Fundamental theorem of Arithmetic
 - Greatest common divisor, least common multiple
 - Concepts of integers, rationals, and real numbers: what integers, fractions, and decimals are; how operations on whole numbers extend to integers and rational numbers and the mathematics that underlies the procedures; and the behavior of units under the operations.
 - Relationship between fractions, decimals, and percents.

- Proportional Reasoning
- Irrational numbers, Properties of Real numbers

C. Rationale: Using middle school mathematics curricula as motivation for in-depth study of the mathematical concepts will connect the course content to the middle school classroom. Teachers will appreciate that what they are learning is relevant to their teaching. Teachers must experience standards-based teaching and learning in order to understand how to implement it. A variety of strategies and methods of instruction should be used to model effective teaching of mathematics.