

Program Cover Document --- MAT 316: Probability

I. Basic Course Information

MAT 316—Probability is primarily a sophomore/junior level course. It is a required course for MATT and MATC majors, and an option for MATA majors and both Mathematics and Statistics minors. It is scheduled for two 80-minute meetings each week. Its prerequisite is MAT 200—Discrete Mathematics and corequisite is MAT 229—Calculus C.

II. Learning Goals

The primary goal of MAT 316: Probability is to provide students with an introduction to the branch of mathematics that has done more to shape the modern view of the world than any other. Students will be introduced to the basic methods of modeling uncertainty, for both discrete and continuous variables, for both univariate and bivariate situations. It serves as a preparatory course for STA 410: Mathematical Statistics, one of the department's required courses for MATC students.

Probability theory is the study of uncertainty, which is an idea common to all areas of human investigation. It plays a key role today in our understanding of the atomic world (quantum mechanics), the living world (natural selection) and socioeconomic interactions (game theory). Probability theory, since its inception in an exchange of letters between Pascal and Fermat, has done more to shape the modern view of the world than any other branch of mathematics, and today reigns triumphant even in the practice of the social sciences. Probability theory leads to unquestionable successes in pointing out regularities in human affairs when people are considered in large groups, for example by insurance companies or when game theory establishes a model for the probabilistic behavior of the reasonable individual.

The Probability course is designed to meet the needs of all the students in the class. The course will give all the necessary background in probability needed for statistics majors and other students to go on to STA 410: Mathematical Statistics. At the same time, Probability is a required course for future secondary school mathematics teachers, and meets the guidelines of the Conference Board of Mathematical Sciences to insure that a student in the teacher preparation program will have a strong foundation in probability. Also, probability is the main subject of the first actuarial exam, and students planning to pursue an actuarial career should be prepared for the first actuarial exam after completing this course.

Topics are chosen to give students the basic knowledge to build mathematical models of the real world. Counting rules will be used to derive models for discrete variables, including the uniform, Bernoulli, binomial, geometric, negative binomial, hypergeometric, and Poisson distributions. Continuous variables will be modeled with distributions having a variety of shapes, including the uniform, exponential, gamma, beta, and normal distributions. Students will learn the relationships between these variables, as well as methods for getting distributions of sums of variables, including the Central Limit Theorem. The relationships between two variables will also be explored using multivariate distributions. Simulation techniques will also be learned.

An additional learning goal of Probability is the further development of both a student's ability to assess the reasonableness of answers and a student's ability to read, write, and understand proofs. Students will also learn to evaluate integrals and sums by using probability functions.

III. Student Assessment

This course is intended to be highly homework intensive. Weekly problem and proof assignments will constantly provide students with opportunities to show their understanding of the material. At the same time students will receive weekly feedback on their work and their progress. A combination of computing assignments, quizzes and tests throughout the course will provide further valuable information both for the instructor and the individual students.

Assessment of the success of Probability in meeting its learning goals will be done through a combination of student performance in the course and in their subsequent use of probability in higher-

level courses such as mathematical statistics. Students' success on the first actuarial exam will also provide information on the success of the course.

IV. Learning Activities

The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, computer assignments, quizzes, tests and final exam.

Departmental Course Syllabus --- MAT 316: Probability

Introduction: A typical syllabus for Probability follows this sheet. Any syllabus for Probability should include the points listed below (the required course requirement sections).

I. Basic information on course and instructor

- A. Purpose statement: Probability introduces students to the fundamental methods of modeling uncertainty using both discrete and continuous models, for one and two variables. Students will learn to choose appropriate mathematical models for real-world situations, and to simulate models that are too complex to evaluate analytically. The course should develop a student's ability to read and do proofs, and to determine the reasonableness of answers. Since probability models the real world, it is often possible for students to come very close to guessing answers before they derive them, and if their answers do not verify their intuition, they should realize this. The course should provide a firm foundation in probability for students taking mathematical statistics and the first actuarial exam.
- B. Course description: An introduction to probability. Topics include mathematical models, sample spaces, conditional probability, discrete and continuous distributions, expected values, moment-generating functions, Central Limit Theorem, multivariate distributions, marginal distributions, and simulation.
- C. Course prerequisite: MAT 200. Course corequisite: MAT 229.

II. Learning goals

- A. Content goals: Students will gain proficiency with many basic topics in probability. The course will introduce students to the definition of probability and proofs using this definition, as well as the basic discrete and continuous probability distributions and their associated expected values, including means, variances, and moment-generating functions. They will learn relationships between variables, and how to evaluate sums and integrals by using probability functions. Models that are too complex to do analytically will be simulated.
- B. Performance goals: At the completion of the course, students should demonstrate competence with probability concepts. They should be able to find probabilities by using counting rules, as well as identifying the situations modeled by discrete random variables. They should know the shapes modeled by the continuous random variables. For all random variables they should be able to find the means, variance, moment-generating functions, and other expected values. They should know the relationships between the different variables and how to find distributions of sums, using both moment generating functions and the Central Limit Theorem. They should be able to recognize when they can evaluate sums and integrals by using probability functions. They should be able to do proofs using the definitions of probability, expected values, and other probability concepts. Finally, they should be able to simulate problems that cannot be done analytically, and determine the reasonableness of all numeric answers.

III. Student assessment

- A. Assessment Plan: This course is intended to be highly homework intensive. Weekly problem and proof assignments will constantly provide students with opportunities to show their understanding of the material. At the same time students will receive weekly feedback on their work and their progress. A combination of computing assignments, quizzes and tests throughout the course will provide further valuable information both for the instructor and the individual students. A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.

- B. Rationale: Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.
- C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to use their knowledge in the future and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

- A. Summary of learning activities: The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, quizzes, tests and final exam.
- B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester. As a general rule, it is expected that each of the major topics of basic probability rules, discrete variables, continuous variables, and multivariate random variables will be given equal emphasis.
- C. Rationale By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of probability and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students receive continual regular feedback on their work.