Math 405: Topology
Departmental Course Syllabus

The study of topology can be approached from several different viewpoints; differential, point-set theoretic or algebraic. A complete treatment of any one of these aspects is beyond the capability of a one semester undergraduate course. However, an understanding of topology gives students an appreciation for the underpinnings of all modern mathematics and is essential for any student continuing on to graduate school. This course, therefore, will serve as an introduction to the principle concepts common to the three areas and an exploration of the techniques of at least one of these. This will allow the students to see some of the deeper connections between topology and other branches of mathematics, in particular, algebra and analysis, which are the focus of much of the undergraduate curriculum.

Abstract algebra is a prerequisite for the course. Since topological invariants tend to be algebraic, this is appropriate.

Learning Goals

Content

Students in this course will gain acquaintance various aspects of topology and its connections to analysis and algebra. This will include a thorough treatment of those aspects of point-set topology that lay the foundation for graduate study; i.e. continuity, compactness, connectedness and separation axioms. Also, this course will introduce students to manifold theory, including the classification of two-dimensional manifolds. The student will understand the role of topological invariants in the classifying topological spaces. This will lead to the classification of the surfaces. Lastly, students will see categorical relationships between topological objects and algebraic objects.

Many students will enter the course with the misconception that the intuition that they developed studying Euclidean spaces generalizes to other topological settings. To confront this, students will be introduced to standard topological counterexamples. Students will come to understand the subtleties inherent in topological definitions.

Performance

The successful topology student will be able to do all of the following:

1. Give proofs of theorems involving point-set theoretic concepts like continuity, compactness, and connectedness.
2. Develop counterexamples to common topological misconceptions.
3. Compute basic invariants.
4. Describe the classification of the surfaces and discuss the current state of knowledge of the classification other collections of topological spaces.

Assessment

Students will be assessed through graded homework assignments, oral communication, and in class and final examinations. The homework assignments and examinations will directly assess the performance goals listed above. Formal and informal oral communication provides students with an alternative route for demonstrating competence with one or more of the performance goals and for developing geometric intuition interactively with fellow students and the instructor.

Homework assignments will provide students with opportunities to attempt lengthier, more challenging problems than is possible on an examination as well as offering students practice at exam-style problems. Examinations enable the professor to assess the knowledge an individual student has readily available.

Learning Activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance at lectures offered by the instructor and other students, written homework assignments, oral presentations, participation in classroom discussions, and group or individual projects.