**Program Cover Document - MAT 426: Partial Differential Equations**

## I. Basic Course Information

MAT 421: Partial Differential Equations is a 1 course unit course. It has two 80-minute meetings each week. Partial Differential Equations is primarily a junior/senior level course. It is designed as a bridge from the foundational Ordinary Differential Equations course to more advanced topics in applied mathematics.

Many physical phenomena can be described by partial differential equations. In this course the initial focus will be on standard methods for determining solutions to the classic equations from physics – the heat equation, wave equation, and Laplace's equation. In developing the solutions to these PDEs we will need to study topics that were not considered in MAT 326, including Fourier Series and transform methods. From these simple models and solution techniques the class will move to consider more general equations, where numerical approximations of solutions will be necessary. The course will be a mixture of theory and computation, with a focus on orthogonal functions and the associated classical theory.

*Course Prerequisite*: MAT 326: Differential Equations.

*Course Description (for Bulletin):* The course is a mixture of theory and computation, with a focus on orthogonal functions and the associated classical theory. It begins with the study of the standard methods for determining solutions to the classic equations from physics using Fourier Series and transform methods, and ends with more general equations where numerical approximations of solutions will be necessary.

## II. Learning Goals

This course will expose students to a variety of differential equations that arise from physical situations. At the end of the course students will be able to:

1. Recreate the derivation of equations from physical principles
2. Utilize techniques developed in the course to determine solutions to different types of equations
3. Incorporate skills mastered in the prerequisite Differential Equations course in the construction of solutions
4. Analyze solutions obtained to assess whether the answers make sense from a physical perspective
5. Use computer software to produce graphs of solutions
6. Approximate solutions to PDEs using numerical techniques

Given that this is a 400 level course, students will improve their ability to work independently on multi-step problems, and students will be exposed to the underlying theory of partial differential equations, including proofs of standard results.

## III. Learning Activities

Learning activities may consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend on the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals.

## IV. Student Assessment

Students will receive feedback on their work through either homework assignments, projects, or examinations.

## V. List of Major Course Topics

The following list of topics will be covered in the course. Items in the right column are optional, however it is expected that the instructor will choose at least two of these topics for inclusion in the course.

| **<u>Required topics</u>** | **<u>Optional topics</u>** |
| --- | --- |
| 1. Introduction to Partial Differential Equations | |
| a. Examples, solutions | |
| b. Ill posed vs well posed | |
| c. Classification of Partial Differential Equations | |
| 2. Fourier Series | |
| a. Inner Product, Norm, Orthogonal Functions | |
| b. General Fourier Series | |
| c. Fourier Cosine and Sine Series | |
| | d. Convergence |
| | 3. Method of Characteristics, Conservation Laws |
| | 4. Change of Variables, Similarity Solutions |
| 5. Separation of Variables | |
| 6. Heat Equation in rectangular coordinates | |
| a. Derivation | |
| b. Solution to one dimensional homogeneous equation by separation of variables | |
| c. Nonhomogeneous case | |
| 7. One Dimensional Wave Equation in rectangular coordinates | |
| a. Solution by separation of variables | |
| | b. d’Alembert solution |

8. Laplace Equation in rectangular coordinates
    a. Solution by separation of variables
    b. Maximum Principle/ Mean Value Property

9. Uniqueness theorem(s), continuous dependence on data

10. Method of Frobenius

11. Special Functions and Sturm Liouville Problems
    a. Bessel Functions, Bessel Series
    b. Legendre Polynomials

12. PDEs in Polar and Spherical Coordinates

13. Numerical Methods
    a. Finite Differences for Laplace Equation, Heat Equation
    b. Implicit Methods
    c. Crank Nicolson

14. Green's Functions

15. Laplace Transforms and/or Fourier Transforms (at least one should be covered)

16. Method of Images

17. Fredholm Alternative

18. Dirichlet Principle/ Calculus of Variations

19. Survey of equations:
    Elasticity,
    Navier-Stokes
    KdV
    Allen-Cahn
    Maxwell
    Schrodinger
    Ginzburg-Landau