I. Learning Goals

In 2001, the Conference Board of Mathematical Sciences, in conjunction with the nation’s two primary national mathematical organizations, the American Mathematical Society and the Mathematical Association of America, published a report entitled, *The Mathematical Education of Teachers.* This report was updated in 2012 and in *The Mathematical Education of Teachers II,* as in the earlier document, they emphasize that prospective high school mathematics teachers need courses “with a primary focus on high school mathematics from an advanced viewpoint.” In fact, their first recommendation is that “Prospective teachers [of all levels] need mathematics courses that develop a solid understanding of the mathematics they will teach.” The content and performance goals for MAT 497 directly address this recommendation.

In MAT 497, teacher candidates will build on what they have learned in their undergraduate mathematics courses, MTT 380, and MTT 390 to explore the inherent coherence of the mathematics of high school and middle school and the structure of mathematical ideas from which the high school and middle school curriculum is derived. Prospective teachers will examine connections between middle school and high school mathematics as well as connections between high school and college mathematics. A teacher who is aware of and understands these connections will be better prepared to teach in a manner consistent with state and national Standards.

MTT 380, MAT 305, MAT 351, and at least one other 300-level course are prerequisites.

II. Student Assessment

Students will receive regular feedback on their work through the assignment of homework, written and oral communications, group and/or individual projects and/or explorations, as well as examinations. Through this feedback, students will be able to see and correct their own misunderstandings and improve their performance. The impact and success of the MAT 497 course on the Mathematics Secondary Education majors will be assessed via the in-class assessments described above and through the success of their senior student teaching semester.

III. Learning Activities

In MTT 380 and 390, teacher candidates have learned a variety of teaching strategies appropriate for teaching mathematics to middle and high school students. They have learned about state and national standards for K-12 mathematics with a focus on middle and high school, and how to implement these standards in the classroom.

In the MAT 497 course, they build on that knowledge to explore the middle school and high school curriculum in depth with an emphasis on the connections between middle school, high school, and the college mathematics courses they have taken. Students must experience standards-based teaching and learning in order to understand how to implement it, as mentioned in the performance goals. Learning activities will consist of a combination of lectures, explorations, group work, participation in class discussions, readings, homework assignments, and group or individual projects. It is important that a variety of strategies and methods of instruction are used to model effective teaching of mathematics.

This study of middle school and high school mathematics will continue in their spring practicum and capstone. The foundation they build in MAT 497 will help prepare them for this experience.
SUGGESTED SYLLABUS FOR COURSE INSTRUCTORS

COURSE TITLE & NUMBER: MAT 497, Topics in Secondary Mathematics from an Advanced Viewpoint

PROFESSOR:

OFFICE ADDRESS:

EMAIL ADDRESS:

OFFICE HOURS:

PREREQUISITES: MTT 380, MAT 305, MAT 351, and at least one other 300-level course are prerequisites.

Course Description and Purpose
This is a mathematics course, but it is aimed at mathematical knowledge, perspectives, and practices that will support the work of those who teach mathematics at the secondary level. Teaching mathematics effectively at the middle and high school level requires that teachers have a deep and profound understanding of the mathematics comprising the K-12 curriculum. The main goal of this course is to enable prospective mathematics teachers to explore and understand, from an advanced perspective, the mathematics that they will teach. In addition, important connections between college level mathematics and all levels of the K-12 curriculum will be emphasized. A teacher’s mathematical knowledge needs to be comprehensive, deep, meaningful, and connected. It is expected that students will have completed the prerequisite courses in which proofs were required and algebraic structures explored. The term “from an advanced viewpoint” means that high school mathematical ideas are examined from a perspective appropriate for college mathematics majors and the course makes use of the knowledge and sophistication the student has gained in other courses. MTT 380, MAT 305, MAT 351, and at least one other 300-level course are prerequisites.

4th Hour
In this class, the deep learning outcomes associated with TCNJ’s 4th hour are accomplished by a series of rigorous educational assignments that extend beyond the typical scheduled class time. These include additional out-of-class reading, writing, and homework problems.

Learning Goals
Teacher candidates will engage in a thorough study of the K-12 mathematics curriculum. Through this process, they will develop a rich understanding of the processes and algorithms found in secondary mathematics and discover purposes behind the symbols and techniques. Many students may enter the course with the misconception that secondary mathematics is all about computation and following or executing procedures. Through careful consideration of many types of problems and connections among ideas, students will come to understand that mathematics is much more than computation, and that there is often more than one approach to solving any problem.

Performance goals
By the completion of the course, the successful student will be able to demonstrate all of the following:
• Deeper understanding of the concepts, processes and algorithms, and the purposes behind them, found in secondary school mathematics.

• An improved ability to communicate mathematical ideas appropriately using the language of mathematics.

• An improved ability to reason mathematically and understand the reasoning of others.

• A willingness and ability to solve various types of mathematical problems using appropriate strategies.

• Knowledge of the relationship of mathematics to other subjects, its applications in a wide range of settings, relationships of ideas studied in school to ideas students may encounter in later study, and relationships within K-12 mathematics itself.

• Knowledge of the history, structure, and application of secondary mathematics.

• Increased understanding of national and state standards.

Course Materials:
Selected readings from:


Articles from: Mathematics Teacher & Mathematics Teaching in the Middle School available online from the library.

Course Requirements and Grading
Students will be assessed and receive regular feedback on their work through the assignment of homework, written and oral communication, group and/or individual projects and/or explorations, and in-class examinations. A syllabus should clearly describe the schedule for these assessment tools, the criteria that will be used to evaluate student performance, and how grades will be calculated.

Course Topics:
The instructor will cover the first 4 topics listed below and choose from among the topics listed in 5, depending on the students’ interests, needs, and backgrounds. A reasonable amount of material would be about 6 of the following topics, explored in depth.

1. Real Number System:
   Rational and irrational numbers, periods of periodic decimals, distributions of various types of real numbers

2. Complex Number System:
Complex plane, geometry of complex numbers

3. Theory of Equations:
   Equality, equivalence and isomorphism, algebraic structures and solving equations, solving inequalities

4. Congruence and Similarity:
   Congruence and similarity transformations, symmetry, traditional congruence

5. Trigonometry, Functions and Modeling, Integers and Polynomials, Data Analysis, Measurement

Students will examine the fundamental concepts of the high school mathematics curriculum including: number, algebra, geometry and functions. Students will explore connections and answer questions such as:

   How do topics from abstract algebra such as solvability of equations, and formulas for solving polynomial equations connect to the high school mathematics curriculum?

   How do topics and applications from linear algebra show up in high school algebra?

   How do congruence transformations currently fit into the traditional high school Euclidean Geometry curriculum?

   How does modeling applied problems using polynomial, exponential, and trigonometric functions fit into the high school mathematics curriculum?

### Example Course Schedule*

<table>
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<tr>
<th>Dates</th>
<th>Content</th>
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| Week 1 | - Syllabus 
Intro to Concept Analysis, Problem Analysis, and Mathematical Connections |
| Week 2 | Building the Real Number system: The beginning properties |
| Week 3 | Building the Real Number system: Decimal representations |
| Week 4 | Properties of Real Functions |
| Week 5 | Building the Complex Numbers |
| Week 6 | Equality, equivalence, and isomorphism |
| Week 7 | Solving process |
| Week 8 | Congruence transformations |
| Week 9 | Symmetry and traditional congruence |
| Week 10 | Distance and similarity |
| Week 11 | Trigonometry |
| Week 12 | Trigonometric functions and their connections |
| Week 13 | Divisibility properties of Integers and Polynomials |
| Week 14 | Systems of Modular Arithmetic and Number Fields |
| Week 15 | Final exam |

*Dates and Topics are subject to change as needed*
Selected TCNJ Policies
TCNJ’s final examination policy is available on the web:
http://www.tcnj.edu/~academic/policy/finalevaluations.htm

Attendance
TCNJ’s attendance policy is available on the web:
http://www.tcnj.edu/~recreg/policies/attendance.html

Academic Integrity
TCNJ’s academic integrity policy is available on the web:
http://www.tcnj.edu/~academic/policy/integrity.html.

Americans with Disabilities Act (ADA) Policy
TCNJ’s Americans with Disabilities Act (ADA) policy is available on the web:
http://www.tcnj.edu/~affirm/ada.html.