Program Cover Document --- MAT 229: Multivariable Calculus

I. Basic Course Information

MAT 229: Multivariable Calculus will be scheduled for three lecture periods: two periods of 80 minutes length and one one-hour meeting period. The prerequisite for the course is MAT 128.

After an introductory section on the geometry of three-dimensional space, we study derivatives of functions of several variables, integrals in dimensions two and three, and Stokes' Theorem (a higher dimensional analog of the Fundamental Theorem of Calculus) in dimensions two and three. Upon completion of the course, we expect students to show competence with the ideas of calculus and its calculations, to understand how to apply calculus to solve real-world problems, to exhibit an improved ability to describe a real-world problem mathematically, to have an increased mathematical maturity, and to have an improved ability to read, write, and understand mathematics.

II. Learning Goals

Multivariable Calculus is a continuation of Calculus A and B, which treat single-variable calculus. The subject matter is part of the foundation that many higher-level courses in mathematics, science, and engineering are built upon. From a practical point of view, a multi-variable approach is essential, as there are few applications with only one variable. As "dimension" is the number of variables, students in this course see for the first time calculus linked with geometry in dimensions two and three.

In Multivariable Calculus, students will gain exposure to both the theoretical and applied aspects of calculus. By working on many real-life problems, students will gain an appreciation for the practical applications of calculus. Simultaneously, their mathematical maturity will be built up through the presentation of theory, and the expectation of a higher level of abstraction and reasoning than has previously been demanded in their mathematics courses. They are also exposed to the concepts and techniques of problem solving through individual and group work on the exercises.

Multivariable Calculus fosters a deeper understanding of the high school geometry that many of our students will teach. Seeing the theorems from single-variable calculus in a more general context develops deeper understanding of the single-variable calculus. Multivariable Calculus also serves as a bridge course between the first year courses and the upper level college mathematical curricula, which build on it both in terms of specific content, and in terms of general mathematical maturity.

III. Student Assessment

Students will receive regular feedback on their work through the assignment of homework, quizzes, student presentations and examinations. Through this feedback, students will be able to see and correct their misunderstandings and improve their performance. Student performance on these assessment instruments and the performance of students in their future courses such as Advanced Calculus, Geometry, Complex Analysis, and Real Analysis will be used to assess the success of Multivariable Calculus in achieving its learning goals and its contribution to the fulfillment of the MATA, MATT, and MATC program goals. Peer reviews and student evaluations will also be used to evaluate the course.

IV. Learning Activities

Learning activities will consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend upon the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals. These learning activities are typical of the learning activities in the MATA, MATT, MATC programs. By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of the concepts of calculus and contribute to the learning goals of these programs.

Departmental Course Syllabus Guide -- MAT 229: Multivariable Calculus

Introduction: A typical syllabus for Multivariable Calculus follows this sheet. Any syllabus for Multivariable Calculus should include the points listed below.

Basic information on course and instructor

A. Purpose statement: Multivariable Calculus is a foundational course for the mathematics, science, and engineering curricula. It introduces students both to the computation of multi-variable calculus, and to the higher level of abstraction of upper-level courses. Multivariable Calculus should inspire mathematical curiosity and interest in students, and should enhance their understanding of geometry.

B. Course description: A course in multi-variable calculus. The course will cover both the theoretical and applied aspects of Calculus.

C. Course prerequisites: MAT 128.

II. Learning goals

A. Content goals: The choice of topics covered and their emphases should be based upon the attached recommended topics list.

B. Performance goals: At the completion of the course, students should show competence with the ideas of multi-variable calculus and its calculations, to understand how to apply calculus to solve real-world problems, to exhibit an improved ability to describe a real-world problem mathematically, to have an increased mathematical maturity, and to have an improved ability to read, write, and understand mathematics.

III. Student assessment

A. Assessment plan: Students will receive regular feedback on their work through the assignment of homework, quizzes, student presentations and examinations. A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.

B. Rationale: Students need to be able to use calculus correctly in their future courses. Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.

C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to display their knowledge of calculus in the future and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

A. Summary of learning activities: Learning activities will consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend upon the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals.

B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester.

C. Rationale By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of the concepts of calculus and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students receive continual regular feedback on their work.

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Multivariable Calculus Topics List

All listed topics are to be covered, except those designated "optional". Topics in bold should be covered in depth. The chapter numbers correspond with Stewart's *Calculus: Early Transcendentals* book.

Chapter 12: Vectors and the Geometry of Space (2 weeks)

12.1 Three-Dimensional Coordinate Systems

12.2 Vectors

12.3 The Dot Product

12.4 The Cross Product

12.5 Equations of Lines and Planes

12.6 Cylinders and Quadric Surfaces

Chapter 13: Vector Functions (1 week)

13.1 Vector Functions and Space Curves

13.2 Derivatives of Vector Functions

13.2 Integrals of Vector Functions (optional)

13.3 Arclength for a Space curve *

13.3 Curvature for a Space curve (optional)

13.4 Motion in Space: Velocity and Acceleration (optional)

*recommended that this topic be postponed and covered with the material in Chapter 16

Chapter 14: Partial Derivatives (3.5 weeks)

14.1 Functions of Several Variables

14.2 Limits and Continuity

14.3 Partial Derivatives

14.4 Tangent Planes and Linear Approximations

14.5 The Chain Rule (including Implicit Differentiation)

14.6 Directional Derivatives and the Gradient Vector

14.7 Maximum and Minimum Values

14.8 Lagrange Multipliers

Chapter 15: Multiple Integrals (3.5 weeks)

15.1 Double Integrals over Rectangles

15.2 Double Integrals over General Regions

15.3 Double Integrals in Polar Coordinates

15.4 Applications of Double Integrals – Moments and Center of Mass (optional)

15.6 Triple Integrals

15.9 Change of Variables in Multiple Integrals

15.7 Triple Integrals in Cylindrical Coordinates

15.8 Triple Integrals in Spherical Coordinates

Chapter 16: Vector Calculus (4 weeks)

16.1 Vector Fields

16.2 Line Integrals

16.3 The Fundamental Theorem for Line Integrals

16.4 Green's Theorem

16.5 Curl and Divergence

16.6 Parametric Surfaces and their Areas

16.7 Surface Integrals

16.8 Stokes' Theorem

16.9 The Divergence Theorem