Program Cover Document - MAT 330: Mathematical Biology

I. Basic Course Information

This course provides an introduction to the interdisciplinary field of mathematical biology. Like other branches of applied mathematics, mathematical biology utilizes mathematical and computational tools to try to answer questions that arise in the real world. Traditionally speaking, mathematics was mostly applied to fields like physics and engineering, while biology operated as a discipline quite separate from mathematics. However, there are many biological questions that are difficult or impossible to answer in the laboratory. Therefore more recently, mathematical techniques have become increasingly important in the biological sciences.

In this course, we will focus on building and analyzing dynamic mathematical models (models that study how processes change in time) to understand the behavior of different biological systems. This course will focus on a variety of topics in population biology, physiology and the biomedical sciences such as single and competing species models, pharmacokinetic models of drugs and toxins, enzyme reaction kinetics, epidemiology, infectious diseases and cancer. We will use mathematical tools like difference equations, differential equations, linear algebra and nonlinear analysis to study these biological processes. Further, computer software packages, such as MATLAB, will be used to implement these mathematical models in search of answers to biological questions. When possible, numerical simulations of the models will be run in MATLAB to compare model outputs to experimental data.

Although the course will focus on building and analyzing mathematical models, the context and applicability of these models to biology will be an important theme of the course. Therefore, while no prior biology knowledge is required for the course, an interest in biology is strongly recommended.

II. Learning Goals

The primary goal of this course is for students to learn to think like mathematical biologists. This requires them to master new mathematical techniques, understand when and how to apply the mathematics in the context of real-world biological problems, use technology to support the mathematics, and have the ability to draw biological conclusions from mathematical results. By the end of the course, students will be able to:

1. Formulate difference and differential equation models of biological systems that are motivated by biological knowledge.
2. Understand how to solve difference and differential equation models of biological systems.
3. Implement models using MATLAB and run computer simulations of the model.
4. Analyze the mathematical results obtained from a model, interpret those results in a biological context, and make biological predictions using the model output.

III. Learning Activities

In-class learning activities include lectures, discussion, group work as well as introductory training in MATLAB. Outside of the classroom, students are expected to do a significant amount of individual work, and in some cases group work, to achieve the learning goals. This work includes the following:

1. Graded homework assignments (written and computer) regularly spaced throughout the semester.
2. At least one independent examination during the semester.
3. A minimum of one significant group project, preferable as a course-culminating final project.
4. A final course evaluation that meets the following criteria.
    a. If a final group project is the final evaluation for the course (that is, no final exam will be given), then the final project must include a significant component that is completed by each individual student in the group (a portion not done by the entire group).
    b. Alternatively, a final examination can be given.

IV. Student Assessment

Students will receive regular feedback on their work through a combination of homework, projects and examinations. Regular textbook-based homework assignments will be used to assess student understanding of the model formulations, mathematical techniques, and computational methods presented in class. Homework assignments and projects will also be used to assess the students' ability to develop mathematical models, and use MATLAB to implement the models and run model simulations.

Project(s) will be assigned during the semester to assess how effectively students can assimilate the various techniques taught in the class to solve larger, more complex biological problems. Formal documentation of the project and its results are important in assessing the student's thinking and writing skills as well as his/her ability to use a mathematical model to draw biologically-plausible conclusions.

Examinations, as detailed in Section III, will be used to assess overall understanding of the course material. The student must demonstrate the ability to think critically about the course content and to integrate various techniques to answer biological questions.

MAT 330: Mathematical Biology
Departmental Course Syllabus

I. Basic Information on Course and Instructor

A. Purpose Statement: Mathematical biology, like other branches of applied mathematics, utilizes mathematical and computational tools to try to answer questions that arise in the real world. Traditionally speaking, mathematics was mostly applied to fields like physics and engineering, while biology operated as a discipline quite separate from mathematics. However, there are many biological questions that are difficult or impossible to answer in the laboratory. Therefore more recently, mathematical techniques have become increasingly important in the biological sciences.

B. Course Description: In this course, we will focus on building and analyzing dynamic mathematical models (models that study how processes change in time) to understand the behavior of different biological systems. This course will focus on a variety of topics in population biology, physiology and the biomedical sciences such as single and competing species models, pharmacokinetic models of drugs and toxins, enzyme reaction kinetics, epidemiology, infectious diseases and cancer. We will use mathematical tools like difference equations, differential equations, linear algebra and nonlinear analysis to study these biological processes. Further, computer software packages, such as MATLAB, will be used to implement these mathematical models in search of answers to biological questions. When possible, numerical simulations of the models will be run in MATLAB to compare model outputs to experimental data.

Although the course will focus on building and analyzing mathematical models, the context and applicability of these models to biology will be an important theme of the course. Therefore, while no prior biology knowledge is required for the course, an interest in biology is strongly recommended.

C. Course Prerequisite and Corequisite: MAT 128 is a prerequisite, and MAT 205 is a corequisite. An interest in biology is recommended for MAT 330. MAT 128 and BIO 201 are prerequisites for BIO 330.

II. Learning Goals

A. Content Goals: Students will learn to think like mathematical biologists. This requires them to master new mathematical techniques (solutions to dynamical systems, both discrete and continuous), to understand when and how to apply the mathematics in the context of real-world biology problems, to use technology to support the mathematics, and to be able to draw biological conclusions from mathematical results.

B. Performance Goals: Upon completing the course, students should demonstrate competence with the ideas of developing mathematical models (difference and differential equations) of biological phenomenon. Further, students should be competent at investigating solutions of these models using analytical and qualitative approaches, as well as computer software (MATLAB). Students should also be able to understand the biological assumptions that are made during the model development process, and be able to interpret the model's results in the context of the biological problem under study.

III. Learning Activities

A. Summary of Learning Activities: Learning activities will consist of a combination of lectures, group work, homework problems, and student projects. While the precise choices and arrangements of these activities depend upon the individual instructor, this must include:

1. Graded homework assignments (written and computer) regularly spaced throughout the semester.
2. At least one independent examination during the semester.
3. A minimum of one significant group project, preferably as a course-culminating final project.
4. A final evaluation, either in the form of an examination or a final group project with a significant independent component for each student in the group.

B. Calendar and Outline: An outline of course topics and a schedule of assessment tools will be provided to the students. Homework, projects and examinations will be spaced at appropriate intervals throughout the semester. It is expected that each of the major topics will be given roughly equal emphasis during the course of the semester.

C. Rationale: This course will introduce students to a variety of approaches to learning and doing mathematical biology. The chosen learning activities should promote a deeper understanding of how to build, solve, use, and interpret mathematical models of biological phenomenon. An even spacing of assessment tools will ensure that students get timely feedback on their progress toward this learning goal.

IV. Student Assessment

A. Assessment Plan: Students will receive timely feedback on their homework, assignments, projects and examinations. Any grading policy should provide a description of the assessment tools that will be used in the course, as well as a grading formula outlining how the assessment tools will be used to calculate the final grade.

B. Rationale: Through the use of timely feedback on homework, projects and examinations, students will have the ability to correct any misunderstandings they might have of the course content, and thereby have the ability to improve their performance in the course. The chosen assessment tools are similar in manner and scope to the ways in which students will need to use their knowledge of the course content in the future. Therefore, these tools are appropriate to assess the accomplishment of the course's learning goals.

C. Methods and Criteria: The instructor will use the assessment of homework, projects and examinations to evaluate students' accomplishments of the course learning goals.

V. Course Outline

See the attached outline of the major topics to be covered in the course.

List of Major Course Topics

The following list of topics will be covered in the course:

I. Model building process

A. Understanding how to read biological statements
B. Translating biological ideas into mathematics
C. Understanding assumptions made in model design

II. Discrete processes and difference equations

A. Linear difference equations: theory and MATLAB
B. Nonlinear difference equations: graphical methods (cobwebbing), fixed points, stability, linearization
C. Linear systems: solution techniques, connection to linear algebra
D. (Optional) Nonlinear systems: graphical methods (phase plane, nullclines, orbits), fixed points, stability, linearization
E. Applications: population growth (exponential and logistic at minimum), age-structured models, etc.

III. Continuous processes and ordinary differential equations

A. Transitioning from discrete to continuous models
B. Foundational techniques for one-dimensional ordinary differential equations: separation of variables, fixed points, stability, phase line, applications to population growth
C. Theory of linear systems of ordinary differential equations: general solution, fixed points, stability
D. Theory of nonlinear systems of ordinary differential equations: fixed points, linearization, stability, phase plane, nullclines, MATLAB numerical solvers
E. Applications of nonlinear systems: population growth, competition and cooperation models, pharmacokinetics, epidemiology and the spread of diseases (SIR models and their extensions), disease dynamics, cancer, chemical kinetics