Program Cover Document --- MAT 430: Seminar in Dynamical Systems

I. Basic Course Information

MAT 430: Seminar in Dynamical Systems is a 1 course unit seminar course. It has two 80-minute meetings each week. Seminar in Dynamical Systems is primarily a junior/senior level course. It is designed as a bridge from the foundational Ordinary Differential Equations course to more advanced topics in applied mathematics.

Prerequisite: MAT 326 – Differential Equations

Course Description (For Bulletin): This course is an introduction to the study of systems whose states change over time – such systems are called dynamical systems. The subject of dynamical systems lies at the border of pure and applied mathematics, with concepts from many areas of theoretical mathematics (analysis, linear algebra, etc.) being used to study these systems as they arise in a broad range of fields, including physics, biology and chemistry. Three main types of dynamical systems will be studied in this class: discrete-time one dimensional maps, continuous-time one dimensional flows on lines and circles, and continuous-time two dimensional linear and nonlinear flows.

II. Learning Goals

Dynamical systems describe time-varying processes fundamental to many scientific theories, and can model phenomena as diverse as the spread of diseases, the motion of planets, and the outcome of chemical reactions. Rich analytic and geometric theories have been developed to study dynamical systems, and the consequences of these theories have strong real-world implications. In this course, we will focus on both the theory and applications of discrete- and continuous-time dynamical systems.

At the end of this course, students will be able to:

1. Understand and prove some of the underlying theory of discrete- and continuous-time dynamical systems.
2. Utilize the theory of dynamical systems to gain quantitative and qualitative insight into time-varying processes.
3. Draw real-world conclusions about the behavior of time-varying systems in a range of fields.
4. Determine how the behavior of a dynamical system changes as parameters vary, along with the potential consequences of these changes in real-world applications.
5. Recognize how and when technology (such as MATLAB) can be helpful or is in fact necessary to gain insights into the behavior of a dynamical system.

Given that this is a 400 level course, students will improve their ability to work independently on multi-step problems, and students will be exposed to the underlying theory of dynamical systems, including proofs of standard results.

III. Learning Activities

Learning activities may consist of any combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend on the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals. As this is a seminar course, the students will be expected to intensively engage in course material under the guidance of the professor. While this could take many forms, examples include project-based learning, or intensive readings followed by in-class faculty-facilitated activities.

IV. Student Assessment

Students will receive feedback on their work through any combination of homework assignments, quizzes, projects, and examinations.

V. List of Majors Course Topics

One Dimensional Maps: Discrete-Time Dynamical System

Required Topics

- Orbit of 1D difference equations
- Fixed points and their local stability
- Basin of attraction
- Cobweb diagrams
- Periodic points
- Logistic map
- Bifurcations
- Liapunov exponent
- Chaos

Optional Topics

- Applications beyond the logistic map
- Universality
- Renormalization
- Period Three Theorem/Sarkovskii's Theorem
- Fractals

One Dimensional Flows: Continuous-Time Dynamical Systems

Required Topics

- Fixed points and phase portraits on a line
- Linear stability analysis for fixed points
- Applications
- Existence and uniqueness
- Impossibility of oscillations on a line
- Perfect bifurcations
- Imperfect bifurcations and catastrophe
- Flows on a circle and oscillators

Optional Topics

- Potentials
- Numerical solution to 1D flows

Two Dimensional Flows: Continuous-Time Dynamical Systems

Required Topics

- Linear and nonlinear systems
- Solution to linear systems
- Fixed points and their local stability
- Phase portraits
- Trace-determinant plane
- Nullclines
- Limit cycles and Poincare-Bendixson Theorem
- Applications

Optional Topics

- Existence and uniqueness
- Conservative systems
- Reversible systems
- Liapunov functions
- 2D bifurcations
- Lorenz equations

SEMINAR IN APPLIED MATHEMATICS: DYNAMICAL SYSTEMS
MAT 454-01, Fall 2019

COURSE DESCRIPTION

Applied Dynamical Systems: This course is an introduction to the study of systems whose states change over time – such systems are called dynamical systems. The subject of dynamical systems lies at the border of pure and applied mathematics, with concepts from many areas of theoretical math (analysis, linear algebra, etc.) being used to study these systems as they arise in a broad range of fields, including physics, biology and chemistry. Three main types of dynamical systems will be studied in this class: discrete-time one dimensional maps, continuous-time one dimensional flows on lines and circles, and continuous-time two dimensional linear and nonlinear flows. Topics to be explored include: fixed points, local stability, geometric methods, bifurcation theory, global stability, existence of periodic solutions and their stability, chaos, and numerous applications across disciplines. Technology will be utilized to assist with our analysis of dynamical systems, as appropriate. (1 course unit)
Prerequisite: MAT 326

INSTRUCTOR INFORMATION

Instructor: Dr. Jana Gevertz **Email**: gevertz@tcnj.edu

Office: Physics & Mathematics P246 **Phone**: 609-771-3314

Office Hours: Monday 5-6:30, Thursday 10:15-11:45, and by appointment

COURSE INFORMATION

Time: Monday and Thursday, 3:30-4:50

Location: Physics & Mathematics P230

Course Website: This course utilizes the Canvas course management system. General course material, reading assignments, online quizzes, homework assignments, and any other important documents will be posted on Canvas. In order to ensure that you get all course announcements, please do not change the default setting in Canvas; the default setting is such that you get immediately notified via email when I post a course announcement.

4th Hour: In this class, deep learning outcomes associated with TCNJ's 4th hour are accomplished by a series of rigorous educational assignments that extend beyond the typical scheduled class time. These include self-guided readings and online quizzes that must be completed prior to each class meeting.

COURSE MATERIALS

Textbook: *Nonlinear Dynamics and Chaos* by Steven H. Strogatz, 2nd Ed.

Technology: Graphing calculator (without symbolic math capabilities) and a software package of your choice, though I will be using MATLAB. MATLAB can be found in campus computer labs, and is freely available for download onto your personal computer at https://software.tcnj.edu/.

Notebook recommendation: I highly recommend you use a three-ring *binder* instead of a notebook for the course, as handouts will be regularly distributed to guide us through lectures and group work.

COURSE PURPOSE & LEARNING GOALS:

Dynamical systems describe time-varying processes fundamental to many scientific theories, and can model phenomena as diverse as the spread of diseases, the motion of planets, and the outcome of chemical reactions. Rich analytic and geometric theories have been developed to study dynamical systems, and the consequences of these theories have strong real-world implications. In this course, we will focus on both the theory and applications of discrete- and continuous-time dynamical systems. The learning goals of this class are to:

1. Develop some of the underlying theory of discrete- and continuous-time dynamical systems.
2. Utilize the theory of dynamical systems to gain quantitative and qualitative insight into time-varying processes.
3. Draw real-world conclusions about the behavior of time-varying systems in a range of fields.
4. Understand how the behavior of a dynamical system changes as parameters vary, along with the potential consequences of these changes in real-world applications.
5. Recognize how and when technology (such as MATLAB) can be helpful or is in fact necessary to gain insights into the behavior of a dynamical system.

COURSE REQUIREMENTS AND GRADING

The following formula will be used to calculate your final grade. In addition, if your average ends up on the border of two letter grades (within half a point), very good/poor class participation can affect your final grade by one-third a letter grade (up/down). *The instructor reserves the right to change or modify this formula as needed.*

Homework Anticipate submitting assignments once every 3 weeks	15%
Quizzes (Online and in-class) Anticipate 1-2 online quizzes a week, based on required readings	15%
Exam #1 Tentative Date: Thursday 10/3	15%
Exam #2 Tentative Date: Thursday 10/31	15%
Exam #3 Due Date: Monday 11/25	15%
Final Exam Date TBA	25%

Class structure: This course will be taught using a partially “flipped” format. This means that for most content, you will begin the process of learning the material on your own through assigned readings. I will provide you with guiding questions to go along with the readings to give you a sense of the main ideas you should be learning. To follow your readings, there will be a short Canvas quiz that must be done before class, and/or a short open-notes (but not open textbook) quiz to be taken at the beginning of class. Class itself will be a mix of lecture and group/individual work. The lectures will mostly focus on the major ideas, theorems and proofs. The group work will be your opportunity to practice working with the material in the context of examples, counterexamples, applications, and computing exercises. It is highly encouraged that you actively participate in class and that you treat it as a comfortable environment in which to ask questions, answer questions, and make mistakes.

Homework: Homework will be assigned 3-4 times throughout the semester, and therefore each assignment will cover a significant amount of material and should be started well before the due date. Assignments will be announced in class and posted on Canvas. You are responsible for knowing all due dates whether you are in class the day an assignment is given or not. You are encouraged to work on the homework with your classmates, although your final write-up must be your own. Homework will consist of both theory and application problems that are meant to be solved by hand, and problems that are best tackled using the aide of computer software, such as MATLAB. *Note that due to the long nature of the homework assignments, the professor reserves to right to grade only a portion of the assignment*. In this case, the grade on the assignment will be a weighted average of your completion grade (100% if the assignment is completed, independent of correctness) and your score on the graded problems.

Attendance, Lateness and Make-ups: Homework and exams are based on material presented in class, so attendance is integral to learning the course material. Assignments are due at the beginning of class, and quizzes start at the beginning of class. Late assignments will not be accepted, and if you miss a quiz due to lateness, it cannot be made up. If you are sick and must miss class, you must email me in advance for us to arrange a time for you to hand in your homework. If you do not do this, you will get a grade of 0 on the assignment. Make-up exams will only be given in extraordinary circumstances when written documentation of the emergency is provided to me. Details on TCNJs College Attendance Policy can be found at: http://policies.tcnj.edu/policies/digest.php?docId=9134.

Academic integrity: You are expected to know the college's policy on academic integrity, which can be found at http://policies.tcnj.edu/policies/digest.php?docId=7642. While I encourage you to work with your classmates on assignments, each write-up must represent your own work. Cheating on exams in any form will not be tolerated. Other forms of academic integrity violations include finding solutions to homework problems online or in a solution manual and submitting the solution as your own, and using MATLAB code written by a classmate or taken from the internet. These violations are not to be taken lightly, so please refrain from ever representing others work as your own.

Students in need of accommodations: Students with documented needs for in-class accommodations should make me aware of this AS SOON AS POSSIBLE! All documented accommodations will be respected, as specified by the Americans with Disabilities Act Policy (http://policies.tcnj.edu/policies/digest.php?docId=8082).

Background: Success in Dynamical Systems depends on your knowledge of Calculus, Linear Algebra, and Differential Equations. If you are rusty on any of the following topics, please review them:

- *Linear Algebra*: solving linear equations, finding nullspaces, linear independence, span, eigenvalues, eigenvectors
- *Differential Equations (DEs)*: solving separable DEs, finding fixed points, determining stability, drawing one-dimensional phase portraits, existence and uniqueness theorems, linear systems of DEs

TENTATIVE COURSE SCHEDULE (Subject to Change)

One Dimensional Maps: Discrete-Time Dynamical System (7-8 classes)
Motivation; orbits; applications; fixed points; attractors; repellers; cobweb plots; logistic map; basins of attraction; periodic points; bifurcations; chaos; Period Three and Sarkovskii's Theorems

One Dimensional Flows: Continuous-Time Dynamical Systems (6-7 classes)
Flows on a line; fixed points; phase portraits; linear stability analysis; applications; existence and uniqueness; impossibility of oscillations; bifurcations; flows on a circle and oscillators

Two Dimensional Flows: Continuous-Time Dynamical Systems (10-11 classes)
Linear and nonlinear systems, fixed points, local stability, phase portraits, trace-determinant plane, linearization of nonlinear systems, nullclines, hyperbolicity, Hartman-Grobman Theorem, conservative systems, reversible systems, Liapunov functions, limit cycles, Poincare-Bendixson Theorem, bifurcations, Lorenz equations, numerous applications