Program Cover Document – MAT 426: Seminar in Partial Differential Equations

I. Basic Course Information

MAT 426: Seminar in Partial Differential Equations is a 1 course unit seminar course. It has two 80-minute meetings each week. Seminar in Partial Differential Equations is primarily a junior/senior level course. It is designed as a bridge from the foundational Ordinary Differential Equations course to more advanced topics in applied mathematics.

Many physical phenomena can be described by partial differential equations (PDEs). In this course, the initial focus will be on standard methods for determining solutions to classical equations from physics, such as the heat equation, the wave equation, and Laplace's equation. To solve these PDEs, we will need to study topics that are not considered in MAT 326, including Fourier Series and. The class will move from these simple models and solution techniques to considering more general equations, where numerical approximations of the PDEs' solutions will be necessary. The course will be a mixture of theory and computation, with a focus on orthogonal functions and the associated classical theory.

Prerequisite: MAT 326: Differential Equations.

Course Description (for Bulletin): The course is a mixture of theory and computation, with a focus on orthogonal functions and the associated classical theory. It begins with the study of the standard methods for determining solutions to the classic equations from physics using Fourier Series, and ends with more general equations where numerical approximations of solutions will be necessary.

II. Learning Goals

This course will expose students to a variety of differential equations that arise from physical situations. At the end of the course students will be able to:

1. Recreate the derivation of equations from physical principles
2. Utilize techniques developed in the course to determine solutions to different types of equations
3. Incorporate skills mastered in the prerequisite Differential Equations course in the construction of solutions
4. Analyze solutions obtained to assess whether the answers make sense from a physical perspective
5. Use computer software to produce graphs of solutions
6. Approximate solutions to PDEs using numerical techniques
7. Write proofs explaining properties of different equations, such as the uniqueness of solutions

Given that this is a 400 level course, students will improve their ability to work independently on multi-step problems, and students will be exposed to the underlying theory of partial differential equations, including proofs of standard results.

III. Learning Activities

Learning activities may consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend on the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the

learning goals. Due to the highly technical nature of the material, students will be expected to complete intensive individual and/or group work both in and out of class. Students will contribute to discussions and other faculty-led activities in class. Outside of class students will regularly complete substantive assignments which could include both reading and writing components. Faculty will provide students with detailed feedback on their understanding of the material and written communication skills. Depending on the individual instructor, other forms of student evaluation may include individual/group projects, quizzes/exams, reports, or presentations.

IV. Student Assessment

Students will receive regular, detailed feedback on their work through either written assignments, projects, or examinations.

V. List of Major Course Topics

The following list of topics will be covered in the course. Items in the right column are optional, however it is expected that the instructor will choose at least two of these topics for inclusion in the course.

Required topics **Optional Topics**

1. Introduction to Partial Differential Equations

    a. Examples, solutions
    b. Ill posed vs well posed
    c. Classification of Partial Differential Equations

2. Fourier Series

    a. Inner Product, Norm, Orthogonal Functions
    b. General Fourier Series
    c. Fourier Cosine and Sine Series

    d. Convergence

3. Method of Characteristics, Conservation Laws

4. Change of variables, Similarity Solutions

5. Separation of Variables

6. Heat Equation in rectangular coordinates
    a. Derivation
    b. Solution to one dimensional homogeneous equation by separation of variables
    c. Nonhomogeneous case

7. One Dimensional Wave Equation in rectangular coordinates
    a. Solution by separation of variables
    b. d'Alembert Solution

8. Laplace Equation in rectangular coordinates
    a. Solution by separation of variables
    b. Maximum Principle/ Mean Value Property

9. Uniqueness theorem(s), continuous dependence on data

10. Method of Frobenius

11. Special Functions and Sturm Liouville Problems

    a. Bessel Functions, Bessel Series
    b. Legendre Polynomials

12. PDEs in Polar and Spherical Coordinates

13. Numerical Methods
    a. Finite Differences for Laplace Equation, Heat Equation
    b. Implicit Methods
    c. Crank-Nicolson

14. Green's Functions

15. Fourier Transform

16. Laplace Transform for PDEs

17. Method of Images

18. Fredholm Alternative

19. Dirichlet Principle/ Calculus of Variations

20. Survey of equations:
    Elasticity,
    Navier-Stokes
    KdV
    Allen-Cahn
    Maxwell

Schrodinger
Ginzburg-Landau

Approved 5/6/2024

Professor Mizuhara
MAT426
Partial Differential Equations
Fall 2022
1.0 Course Units

Instructor: Matthew Mizuhara
Office: SCP - 211
Email: mizuharm@tcnj.edu

Class Times: **Section 1 –** Tuesday/Friday 2:00 pm – 3:20 pm (P201)
Section 2 – Tuesday/Friday 11:00 am – 12:20 pm (P229)

Office Hours:
Monday 10:00 – 11:30am
Wednesday 9:30 – 11:00am (shared)
By appointment

Communication: The instructor will contact students primarily by Canvas announcement or by e-mail. Students can contact the instructor by Canvas or by e-mail and usually expect a 24 hour response time (during the normal work week). Course materials will be available on Canvas.

Course Schedule: No classes on
Tuesday, Sept 6 (follow Monday schedule)
Tuesday, Oct 11 (Fall Break)
Friday, Nov 25 (Thanksgiving Break)

Textbook: There is no required textbook, but the following are useful companion texts for the course for extra practice

1. Applied Partial Differential Equations by J. D. Logan
2. Introduction to Applied Partial Differential Equations by J. M. Davis
3. Fundamentals of Differential Equations and Boundary Value Problems by R. Nagle, E. B. Saff, and A. D. Snider

Prerequisites: MAT 326: Differential Equations

Technology and technical skill requirements: Students will require access to a computer and/or internet connected device (e.g. tablet) with the following features:

1. Ability to access the course Canvas and Perusall websites
2. Ability to use MATLAB

Masking policy: Masking in class will be optional. Please be thoughtful of the varying comfort levels of your peers. If health conditions in NJ or on campus worsen, we will revisit the issue.

Grading distribution:
Homework + readings = 15%
Quizzes = 15%
Class/Group work = 5%
Exam 1 = 15%
Exam 2 = 15%

Exam 3 = 15%
Final = 20%

Course Description: The course is a mixture of theory and computation, with a focus on orthogonal functions and the associated classical theory. It begins with the study of the standard methods for determining solutions to the classic equations from physics using Fourier Series and transform methods and ends with the study of more general settings where numerical approximations of solutions will be necessary.

Course Purpose: This course serves as a survey of several topics of classical partial differential equations theory. The course will focus on the three canonical equations: the heat equation, wave equation, and Laplace equation. We will learn solution techniques beyond those explored in MAT326, including Fourier Series, orthogonal decompositions, and transform methods and see how they can be applied to a wide range of problems. Simultaneously, we will study the theoretical underpinnings of each equation, introducing powerful notions such as associated energy, maximum principles, and mean value properties. Finally, we will use computational techniques to supplement our understanding of each equation as well as develop numerical approximation techniques to solve more general equations. By the end of the course students will be able to compare each classical partial differential equation and apply a wide range of theoretical and computational techniques to study them.

Learning Goals:

1. Students will be able to identify and solve PDEs using a variety of techniques including substitution, partial integration, techniques from ODEs, and separation of variables.
2. Students will be able to explain the similarities and differences between properties and solution methods of the three classic PDEs.
3. Students will be able to create proofs analyzing properties of PDE solutions.
4. Students will be able to leverage numerical algorithms to solve and approximate solutions of partial differential equations.
5. Students will develop their ability to read, write, and discuss new mathematical ideas independently/collaboratively. Students will become more independent and critical mathematical thinkers and learners.
6. This course satisfies the TCNJ "Quantitative Reasoning" Liberal Learning outcome: apply formal logic or numerical analysis to evaluate a quantitative claim, guide a decision, solve a problem, or answer a question.

Course Expectations:

1. Students are expected to attend and participate in all class sessions by asking and answering questions, as well as contributing to peer group discussions. Students who must miss a class due to extenuating circumstances should notify me as soon as possible. *If you are sick, you should not attend class.*
2. Students may be asked to complete readings or watch supplementary videos **before** class to prepare for questions and discussion. Students will be required to comment on their readings using the Perusall online system.
3. Math is collaborative and we learn best when working in groups of diverse backgrounds and experiences. Students are encouraged to form groups to collaborate and discuss material outside of class.
4. All members of the classroom will contribute to a respectful, welcoming, and inclusive environment for all other members of the course. We all will strive to acknowledge and reduce our biases in the classroom. This aligns with the college's commitment to diversity,

equity, inclusion, access, and belonging (https://diversity.tcnj.edu/campus-diversity-statement/)

5. If anyone's comfort or performance in the class is affected by any classroom or external situations, they should feel safe contacting me to address the issue.

Fourth hour: The fourth hour will consist of group and individual work to gain a deeper understanding of course material.

Homework: Students will be given weekly homework assignments. Solutions must be neatly written (or typed) in complete sentences. Answers found and copied from online are invalid and will not be rewarded credit. Homework problems can be discussed with peers in the class, but solutions must be written individually, in your own words. **Collaborators on homework must be indicated on each homework assignment.** Homework extensions will be considered on a case by case basis if appropriate notice is provided by the student.

Quizzes: There will be quizzes non-exam weeks taking approximately 15 minutes. Anyone arriving late to class or leaving class after the quiz will not receive credit. **Quizzes will always be announced.**

Exams: There will be three in-class exams. The second and third exams will not be cumulative.
Tentative exam dates: Sept 30, Oct 28, Nov 29

Final: There will be a cumulative final exam at the end of the semester. The final exam period is Dec 13-Dec 20. The College Final Examination Policy is available at http://policies.tcnj.edu/policies/digest.php?docId=9136.

Further policies: The TCNJ community is dedicated to the success, safety and well-being of each student. TCNJ strictly follows key policies that govern all TCNJ community members rights and responsibilities in and out of the classroom. In addition, TCNJ has established several student support offices that can provide the support and resources to help students achieve their personal and professional goals and to promote health and well-being. You can find more information about these policies and resources at the "TCNJ Student Support Resources and Classroom Policies" webpage here: https://academicaffairs.tcnj.edu/tcnj-syllabus-resources/.

Students who anticipate and/or experience barriers in this course are encouraged to contact the instructor as early in the semester as possible. The Accessibility Resource Center (ARC) is available to facilitate the removal of barriers and to ensure reasonable accommodations. For more information about ARC, please visit: https://arc.tcnj.edu/.

Overview of course content:

Section 1: Introduction to PDEs
Section 2: Transport Equation
Section 3: Heat Equation
Section 4: Fourier Series
Section 5: Wave Equation
Section 6: Laplace Equation
Section 7: Sturm-Liouville Equations
Section 8: Numerical Methods
Section 9: Greens Functions
Section 10: Fourier Transform

Tentative Timeline (subject to change):

Week 1 : Section 1	Week 2 : Section 2	Week 3 : Section 3
Week 4 : Section 3	Week 5 : Section 4, Exam 1	Week 6 : Section 5
Week 7 : Section 6	Week 8 : Section 7	Week 9 : Section 7, Exam 2
Week 10 : Section 8	Week 11 : Section 9	Week 12 : Section 9
Week 13 : Section 10	Week 14 : Section 10, Exam 3	Week 15 : Review

The instructor reserves the right to change the syllabus policies at any point of the semester