Departmental Course Syllabus --- MAT 301: Number Theory

Introduction: A typical syllabus for Number Theory follows this sheet. Any syllabus for Number Theory should include the points listed below (the required course requirement sections).

I. Basic information on course and instructor

A. Purpose statement: Number Theory introduces students to some of the classical problems in elementary number theory. While learning the subject matter of number theory, the course will also develop a student's ability to reason abstractly, to read mathematics, and to prove theorems. Through the use of challenging problems, the course should also develop a student's problem solving ability and introduce students to the joys of mathematics. The course should also provide a firm foundation in modular arithmetic for students taking Abstract Algebra.

B. Course description: An introduction to number theory. Topics include divisibility, primes, unique factorization, Diophantine equations, congruences, and quadratic reciprocity. Optional topics may include sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves.

C. Course prerequisites: MAT 200. This requirement can be met with CSC 270 and permission of the chair.

II. Learning goals

A. Content goals: Students will gain acquaintance with many basic topics in elementary number theory. Students will learn about primes, unique factorization, congruences, divisibility, Diophantine equations, primitive roots, and quadratic reciprocity. Other optional topics such as sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves may be included at an instructor's discretion.

B. Performance goals: At the completion of the course, students should demonstrate competence with number theory concepts. A successful number theory student should be able to do modular arithmetic and know how to use modular arithmetic to study Diophantine equations. They should know basic procedures to determine whether or not a Diophantine equation has solutions. They should understand the definition of a prime, their central role in arithmetic problems and be knowledgeable about primes in number systems other than the integers. They should also be proficient at finding and working with primitive roots. Students should possess improved reasoning and proof-writing ability and should exhibit a more mature ability with proofs than would normally be expected of a student at the completion of MAT 205. In particular, they should be comfortable with important techniques of proofs such as Fermat's theory of descent.

III. Student assessment

A. Assessment plan: Students will receive regular feedback on their work through the assignment of homework, quizzes, student presentations and examinations. A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.

B. Rationale: Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.

C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to use their knowledge in the future of and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

A. Summary of learning activities: Learning activities will consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend upon the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals. Students should be expected to use appropriate tools, including computers and simple computer programs, as well as concrete models or algorithms, for explorations in Number Theory.

B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester. As a general rule, it is expected that each of the major topics of primes, unique factorization, congruences, divisibility, Diophantine equations, primitive roots, and quadratic reciprocity will be given equal emphasis during the course of the semester.

C. Rationale By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper number theory understanding and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students continual regular feedback on their work.

"Mathematics is the queen of the sciences, and the theory of numbers is the queen of mathematics."
- Karl Friedrich Gauss

"To Think Deeply About Simple Things." - Arnold E. Ross

CATALOG DESCRIPTION

MAT 301/Number Theory: This course is an introduction to number theory. Topics include divisibility, primes, unique factorization, Diophantine equations, congruences, and quadratic reciprocity. Optional topics may include sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves. (1 course unit)
Prerequisite: MAT 200. This requirement can be met with CSC 270 and permission of the chair.

INSTRUCTOR INFORMATION

Instructor: Dr. Thomas Hagedorn

Office: Physics & Mathematics P240

Email: hagedorn@tcnj.edu

Phone: 609-771-3053

Office Hours: M 3:30-4, W 9-10, F 9-10
(Zoom)

Additional Help Session: W: 12:30-1:30 with math major Meghan Ryan.

Mathematical Trivia: 1729, 17, 28, 341, 561, 496, 8128, and 65537 are some of my favorite numbers. Newton, Leibniz, Fermat, Euler, and Gauss are some of my favorite mathematicians.

COURSE INFORMATION

Time: Monday and Thursday2-3:20 in SCP 229

Course Website: This course utilizes the Canvas course management system. General course material, Zoom links, homework assignments, group work quizzes, and any other important documents will be posted on Canvas.

4th Hour: In this class, deep learning outcomes associated with TCNJ's 4th hour are accomplished by activities conducted in the scheduled 4th hour class meeting time.

COURSE MATERIALS

Required Material: Our textbook will be Strayer's *Elementary Number Theory*. It is available from the bookstore. In addition, used copies are available on the internet. Any edition will work.

Required Software/Technology: The course will occasionally use Zoom video for office hours. The link will be available from within our Number Theory Canvas course. If you do not have access to the necessary equipment for this course, TCNJ can provide it for you for the semester. Please contact care@tcnj.edu to help you secure the needed technology free of charge. This is a confidential process.

COURSE DESCRIPTION & LEARNING GOALS

Number theory deals with the study of the integers (0, ±1, ±2, ...). In our course, we view them not solely as units to measure lengths, but as fundamental objects (like atoms in physics) and study the various relationships between them. A few properties we will study include divisibility, factorization, congruences. Prime numbers will soon enter the story and we see that there are an infinite number of primes, and numbers have a unique factorization into primes. We will study how to determine when polynomial equations have integer solutions. This knowledge is fundamental to cryptography and the security of modern-day communications, and we will study this application at the end of the course.

Number Theory is one of the oldest areas of mathematics. Problems in the subject are often easy to state and understand but are often surprisingly difficult to solve. The Greeks posed many number theory problems and solved some of these. Yet, other problems have remained unsolved until this day. Many other problems have been studied in the intervening centuries and number theory continues to be an exciting area of mathematics. The course has the following learning goals. Students will:

- Learn the content of elementary number theory and the many classical problems that have challenged people throughout history; and
- Use problem solving skills to solve math problems, and to prove or disapprove, and salvage if possible, conjectures in number theory; and
- Use computer software to make conjectures and solve problems in number theory; and
- Learn modern cryptography based on number theory; and
- Provide future teachers with a solid foundation in number theory that can be used to provide enrichment in the K-12 curriculum; and
- Improve their ability to reason abstractly, to read mathematics, and to write proofs. It serves as a preparatory course in the department to prepare students for the more abstract and rigorous material found in MAT 305: Abstract Algebra; and
- Most importantly, learn to think like a mathematician. In this course, you'll see how we find and create mathematics by looking at numerical evidence (whether done by hand or computer), conjecturing patterns found in the evidence, and then trying to prove if these conjectures are true. In doing this, you'll hopefully experience both the joys and *frustrations* of doing mathematics, develop perseverance in working on difficult problems, and experiencing the delight in ultimately discovering new mathematical knowledge.

COURSE POLICIES

Class goals: Our classroom is intended to be an equitable and inclusive learning environment. If something about the course is not working for you, please reach out to me so we can fix that.

Class structure: Class will be a mix of interactive lectures and group work. It is highly encouraged that you actively participate in class and treat it as a comfortable environment in which to ask questions, answer questions, and make mistakes. The material we will cover in a class will be posted ahead of time. It is expected that you will read over this material in advance and complete any requested problems.

Website/Email: All course information, announcements, and assignment/test grades will be posted on Canvas. It also contains information that may help you succeed in this course. I'll assume you are keeping up to date with its contents. Please email me with questions as they arise. My goal is to respond to emails with a day, but sometimes other obligations prevent this. If I haven't responded by the next class meeting, please speak to me in person.

Homework, Quizzes, and Group Work: There will be regular homework assignments and quizzes. Students will be asked to work on problems ahead of class and present their work in class. Some homework problems will be turned in and graded. Homework grades will based both on the written homework and work presented in class. Quizzes will be based on the homework and readings.

Tests and Assessment: We will have two tests equally spaced, in the semester, and a final exam. Grades will be calculated using the following formula homework (15%), quizzes (15%), exams (20% each), and a final exam (30%). Exceptional/poor class participation may change one's grade by one-third letter grade. The professor reserves the right to adjust the number or type of assignments, and the grading formula, as needed, to accommodate changes in the course during the semester. Grades will be assigned using the following grading scale, and you can see your current course grade on Canvas.

A: 93 – 100	B-: 80 – 82.99	D+: 67 – 69.99
A-: 90 – 92.99	C+: 77 – 79.99	D: 60 – 66.99
B+: 87 – 89.99	C: 73 – 76.99	F: < 60
B: 83 – 86.99	C-: 70 – 72.99	

Attendance and Late Policy: Homework, quizzes, and exams are based on material presented in class, so attendance during class is integral to learning the course material. But life happens and I encourage you to contact me if there are reasons you are not able to attend class. However, the policy around exams is stricter. Makeup exams will only be given in extraordinary circumstances, and we must discuss this *in person* before the exam. Late night emails will not excuse you from the exam. Details on TCNJs College Attendance Policy can be found here.

Academic integrity: You are expected to know the college's policy on academic integrity, which can be found here. While I encourage you to work with your classmates on assignments, each write-up must represent your own work. Please refrain from representing the work of others, *including solutions obtained online or through AI*, as your own. If something about the way the course is running tempts you to violate our academic integrity policy, then it is likely there is more I can do to support your learning. In that case, please let me know what I can do to make the class work better for you!

Resources for Student Success: The TCNJ community is dedicated to the success, safety and well-being of each student. TCNJ strictly follows key policies that govern all TCNJ community members' rights and responsibilities in and out of the classroom. In addition, TCNJ has established several student support offices that can provide the support and resources to help students achieve their personal and professional goals and to promote health and well-being. You can find more information about these policies and resources at the "TCNJ Student Support Resources and Classroom Policies" webpage at https://academicaffairs.tcnj.edu/tcnj-syllabus-resources/.

Students who anticipate and/or experience barriers in this course are encouraged to contact the instructor as early in the semester as possible. The Accessibility Resource Center (ARC) is available to facilitate the removal of barriers and to ensure reasonable accommodations. For more information about ARC, please visit: https://arc.tcnj.edu/.

While I encourage you to come to my office hours and contact me via email to work through any difficulty you have with the material, I realize that sometimes students want a fresh perspective on the material. I highly recommend you study with your classmates and talk about material with one another. If you find you need more individualized help, you can also visit the Tutoring Center at Roscoe West, Suite 203. There are some great students in the Mathematics & Statistics Department that tutor as well – their hours will be available shortly.

Commitment to Diversity, Equity, Inclusion, Access and Belonging: The TCNJ community is composed of people with diverse backgrounds, perspectives, and experiences. The college's Campus Diversity Statement can be viewed here: https://diversity.tcnj.edu/campus-diversity-statement/.

Liberal Learning Outcomes: MAT 301/Number Theory has a Quantitative Reasoning designation as part of the Liberal Learning program. Quantitative Reasoning courses have the following learning outcomes:

- Students should understand quantitative reasoning so they can respond effectively to claims deriving from quantitative arguments.
- Students will understand how real-world problems and social issues can be analyzed using the power and rigor of quantitative methods while also learning to recognize and articulate the limitations of these methods.
- Students will be able to do all the following: evaluate, interpret, and draw inferences from mathematical models such as algorithms, formulas, graphs, and tables.
- Students will be able to use quantitative methods (such as algebra, geometry, statistics and computation) to solve problems.