Program Cover Document --- MAT 305: Abstract Algebra

I. Basic Course Information

MAT 305: Abstract Algebra is primarily a junior/senior level course. It is scheduled for two 80minute meetings each week. Its prerequisites are MAT 200 and MAT 205. The MAT 200 prerequisite can be met with CSC 270 and permission from the chair.

Abstract Algebra is one of the fundamental disciplines in modern mathematics. Starting in antiquity with everyday practical problems, it developed in the eighteenth and nineteenth centuries into a theoretical discipline unifying fields like number theory, the theory of equations and geometry. Historically, much of the theory in abstract algebra was developed independently by number theorists and by geometers. The outcome of these efforts was a discipline that interprets the foundations of many areas in mathematics.

II. Learning Goals

The primary goal of MAT 305: Abstract Algebra is to provide students with an introduction to one of the fundamental subjects in modern mathematics.   Students will be introduced to the basic algebraic structures of groups, rings, and fields and the abstract notions that unify these diverse mathematical objects.

The Abstract Algebra course is designed to meet the needs of all the students in the class.   The course will give all the necessary background in abstract algebra for students like liberal arts math majors (MATA) who wish to study more advanced topics in abstract algebra or students from the sciences who will use the algebraic structures in their research.   At the same time, Abstract Algebra is a required course for future secondary school mathematics teachers. It has been developed following the guidelines of the Conference Board of Mathematical Sciences to insure that a student in the teacher preparation program will have a strong foundation in algebra.

Topics are chosen to cover historically important problems in algebra, to give an understanding of the subject's historical development and to illustrate the common underlying algebraic structure beneath seemingly unrelated subjects. The basic algebraic structures are groups, rings, integral fields, and fields. Fundamental applications of their basic properties will be seen by students in the arithmetic of the integers, the integers mod n, the rational numbers, the real numbers, the complex numbers and the transformations of geometric objects such as regular polygons.

An additional learning goal of Abstract Algebra is the further development of both a student's abstract reasoning ability and a student's ability to read, write, and understand proofs.   The level of proofs in Abstract Algebra is at a high level and builds upon the techniques of proof that a student has learned in MAT 200 and MAT 205.

III. Student Assessment

This course is intended to be highly homework intensive. Weekly reading and writing assignments will constantly provide the instructor with information on the progress of individual students. At the same time students will receive weekly feedback on their work and their progress. A combination of quizzes and tests throughout the course will provide further valuable information both for the instructor and the individual students.

Assessment of the success of Abstract Algebra in meeting its learning goals will be done through a combination of student performance in the course and in their subsequent use of algebra in higher-level courses such as the senior capstone seminar.

IV. Learning Activities

        The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, quizzes, tests and final exam.

Departmental Course Syllabus --- MAT 305: Abstract Algebra

Introduction: A typical syllabus for Abstract Algebra follows this sheet. Any syllabus for Abstract Algebra should include the points listed below (the required course requirement sections).

I. Basic information on course and instructor

    A. Purpose statement:

        Abstract Algebra is one of the fundamental disciplines in modern mathematics. Starting in antiquity with everyday practical problems, it developed in the eighteenth and nineteenth centuries into a theoretical discipline unifying fields like number theory, the theory of equations and geometry. Historically, much of the theory in abstract algebra was developed independently by number theorists and by geometers. The outcome of these efforts was a discipline that interprets the foundations of many areas in mathematics.

        The arithmetic of the integers, the integers mod n, the rational numbers, the real numbers, the complex numbers as well as the transformations of certain geometric objects are just examples of algebraic structures. The basic underlying algebraic structures groups, rings, integral domains and fields are the abstract notions that unify these diverse mathematical objects.

        Math 305: Abstract Algebra will first provide the students with an introduction to all the basic algebraic structures, groups, rings, integral domains and fields with numerous concrete applications and their basic properties. In the case of groups the symmetries of regular polygons and the groups of symmetries in general will be studied . In the case of rings, the ring of integers and the rings of polynomials over a field, will give the students an understanding of the historical development of the subject. The algebraic similarities between the ring of integers and the ring of polynomials will be emphasized. The construction of the field of rational numbers will be studied. In addition the field of real numbers, the field of complex numbers and the field of integers mod a prime p will be used when studying solutions of polynomial equations.

    B. Course description: An introduction to groups, rings, integral domains, and fields. Additional topics include groups of symmetries, rings of integers, roots of polynomial equations, and construction of the rational numbers.

    C. Course prerequisites: MAT 200 and MAT 205. The MAT 200 prerequisite can be met with CSC 270 and permission from the chair.

II. Learning goals

    A. Content goals: Students will gain proficiency with many basic topics in abstract algebra. The course will introduce students to groups, rings, integral domains, and fields. They will learn about symmetry groups and the roots and factorization of polynomials defined over various rings and fields. They will learn about the construction of various number systems.

    B. Performance goals: At the completion of the course, students should demonstrate competence with abstract algebra concepts. A successful abstract algebra student should be able to work with symmetry groups and understand their usefulness. They should be able to do algebra calculations

involving polynomials over various number systems. They should be comfortable working with unusual groups and rings. They should understand the connection between the abstract systems they have studied and the concrete properties of the number systems they are familiar with.   They should understand the connection between algebra and the solution of problems from other fields of mathematics.

III. Student assessment
  A. Assessment Plan: This course is intended to be highly homework intensive. Weekly reading and writing assignments will constantly provide the instructor with information on the progress of individual students. At the same time students will receive weekly feedback on their work and their progress. A combination of quizzes and tests throughout the course will provide further valuable information both for the instructor and the individual students.   A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.
  B. Rationale: Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.
  C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to use their knowledge in the future and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

  A. Summary of learning activities: The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, quizzes, tests and final exam.
  B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester. As a general rule, it is expected that each of the major topics of groups, rings, and fields will be given equal emphasis.
  C. Rationale   By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of abstract algebra and contribute to the learning goals of these programs.   A regular spacing of assessment tools insures that students continual regular feedback on their work.

Updated: 2/12/2025

MAT 305

ABSTRACT ALGEBRA

SPRING 2025

PROFESSOR: AIGLI PAPANTONOPOULOU   EMAIL ADDRESS: AIGLI@TCNJ.EDU

OFFICE HOURS: MON AND THURSDAY. 4:00-5:00 PM AND BY APPOINTMENT

COURSE DESCRIPTION

Abstract Algebra is primarily a junior/senior level course. It is scheduled for two 80-minute meetings each week. Its prerequisites are MAT 200 and MAT 205. (The MAT 200 requirement can be met with CSC 270 and permission of the chair.)

In this class, the deep learning outcomes associated with TCNJ's 4th hour are accomplished by a series of rigorous educational assignments that extend beyond the typical scheduled class time, these include additional group work as explained below under Course Requirements.

COURSE MATERIALS

ALGEBRA : Pure & Applied, by Aigli Papantonopoulou

COURSE REQUIREMENTS

An assignment will be due every **Thursday** but not collected, instead a quiz will be given in class on the assignment**.** Students are encouraged to join a study group. Study groups should meet at least once a week to work together on the assignments.

COURSE PURPOSE & LEARNING GOALS

The primary goal of MAT 305: Abstract Algebra is to provide students with an introduction to one of the fundamental subjects in modern mathematics.   Students will be introduced to the basic algebraic structures of groups, rings, integral domains and fields and the abstract notions that unify these diverse mathematical objects.

COURSE SCHEDULE

The course will cover Chapters 1, 2, 6, 7, 8 and parts of 10. The first four assignments are in the next attachment. Two in class tests will be given during the semester. The dates are:

Test 1 on **Monday March 3**

Test 2 on **Monday April 14**

Course grades will be determined as follows

Quizzes	**100 pts**
Test 1	**100 pts**
Test 2	**100 pts**
Final Exam	**100 pts**
Total	**400 pts**

There will be **NO MAKE-UPS** for any of the quizzes or tests , and please **NO CELL-PHONES** in class.

SELECTED TCNJ POLICIES

TCNJ's final examination policy is available on the web:
http://policies.tcnj.edu/policies/digest.php?docId=9136

Attendance

Every student is expected to participate in each of his/her courses through regular attendance at lecture and laboratory sessions. It is further expected that every student will be present, on time, and prepared to participate when scheduled class sessions begin. At the first class meeting of a semester, instructors are expected to distribute in writing the attendance policies which apply to their courses. While attendance itself is not used as a criterion for academic evaluations, grading is frequently based on participation in class discussion, laboratory work, performance, studio practice, field experience, or other activities which may take place during class sessions. If these areas for evaluation make class attendance essential, the student may be penalized for failure to perform satisfactorily in the required activities. Students who must miss classes due to participation in a field trip, athletic event, or other official college function should arrange with their instructors for such class absences well in advance. The Office of Academic Affairs will verify, upon request, the dates of and participation in such college functions. In every instance, however, the student has the responsibility to initiate arrangements for make-up work.

Students are expected to attend class and complete assignments as scheduled, to avoid outside conflicts (if possible), and to enroll only in those classes that they can expect to attend on a regular basis. Absences from class are handled between students and instructors. The instructor may require documentation to substantiate the reason for the absence. The instructor should provide make-up opportunities for student absences caused by illness, injury, death in the family, observance of religious holidays, and similarly compelling personal reasons including physical disabilities. For lengthy absences, make-up opportunities might not be feasible and

*are at the discretion of the instructor. The Office of Academic Affairs will notify the faculty of the dates of religious holidays on which large numbers of students are likely to be absent and are, therefore, unsuitable for the scheduling of examinations. Students have the responsibility of notifying the instructors in advance of expected absences. In cases of absence for a week or more, students are to notify their instructors immediately. If they are unable to do so they may contact the Office of Records and Registration. The Office of Records and Registration will notify the instructor of the student's absence. The notification is not an excuse but simply a service provided by the Office of Records and Registration. Notifications cannot be acted upon if received after an absence. In every instance the student has the responsibility to initiate arrangements for make-up work.*TCNJ's attendance policy *is available on the web:* http://policies.tcnj.edu/policies/digest.php?docId=9134*

Academic Integrity Policy

Academic dishonesty is any attempt by the student to gain academic advantage through dishonest means, to submit, as his or her own, work which has not been done by him/her or to give improper aid to another student in the completion of an assignment. Such dishonesty would include, but is not limited to: submitting as his/her own a project, paper, report, test, or speech copied from, partially copied, or paraphrased from the work of another (whether the source is printed, under copyright, or in manuscript form). Credit must be given for words quoted or paraphrased. The rules apply to any academic dishonesty, whether the work is graded or ungraded, group or individual, written or oral.

TCNJ's academic integrity policy *is available on the web:* http://policies.tcnj.edu/policies/digest.php?docId=7642

Americans with Disabilities Act (ADA) Policy

Any student who has a documented disability and is in need of academic accommodations should notify the professor of this course and contact the Office of Differing Abilities Services (609-771-2571). Accommodations are individualized and in accordance with Section 504 of the Rehabilitation Act of 1973 and the Americans with Disabilities Act of 1992.

TCNJ's Americans with Disabilities Act (ADA) policy is available on the web: http://policies.tcnj.edu/policies/digest.php?docId=8082

"The TCNJ community is composed of people with diverse backgrounds, perspectives, and experiences, and the college is committed to diversity, equity, inclusion, access and belonging. The college's Campus Diversity Statement can be

viewed here: https://diversity.tcnj.edu/campus-diversity-statement/." "The TCNJ community is dedicated to the success, safety and well-being of each student. TCNJ strictly follows key policies that govern all TCNJ community members rights and responsibilities in and out of the classroom. In addition, TCNJ has established several student support offices that can provide the support and resources to help students achieve their personal and professional goals and to promote health and well-being. You can find more information about these policies and resources at the "TCNJ Student Support Resources and Classroom Policies" webpage here: https://academicaffairs.tcnj.edu/tcnj-syllabus-resources/.