Program Cover Document - MAT 310: Real Analysis

I: Basic Course Information

MAT 310 is a junior/senior level course that serves as an introduction to Real Analysis. Analysis is broad field of mathematics that is based on concepts students have studied on a very elementary level in calculus. Real Analysis is the branch of this discipline that considers properties and behavior of real-valued functions of a real variable.

The course will be scheduled for two 80-minute lecture periods. Its prerequisites are MAT 200 and MAT 229 (the MAT 200 prerequisite can be met with CSC 270 and permission from the chair). 1 course unit.

II: Learning Goals

The choice of content for MAT 310 serves two goals. First, many concepts studied in MAT 310 are first studied in more elementary calculus courses such as MAT 127, 128, and 229. The more rigorous treatment of those concepts in Real Analysis will give the student a clear understanding of the definition of differentiation and integration, and familiarize the student with the logical unfolding of the fundamental principles of calculus, from a mature point of view. Secondly, a majority of MAT 310 students are mathematics majors or mathematics education majors. The study of Real Analysis will help the student acquire solid background knowledge and maturity that will enable him or her to become an inspiring teacher of calculus and other advanced mathematics, and prepare the student who wishes to continue graduate study in mathematics for advanced work in mathematical analysis.

III. Learning Activities

The specific choice of learning activities will depend upon the instructor, but it is expected that they will consist of a combination of lectures, readings, group work, individual homework, student presentation, tests, and exams. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals. By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of the concepts of Real Analysis and contribute to the learning goals of the specializations in the Department of Mathematics and Statistics.

IV. Student Assessment

It is expected that students will show competence with the ideas of real analysis and thorough understanding of its concepts, will exhibit an ability to think rigorously and abstractly, and will demonstrate an improved ability to read, write, and understand proofs. The specific assessment plan will be at the instructor's discretion but should be primarily based upon a combination of exams, presentations, and homework. Given the

particularly challenging material of abstract concepts and proofs, it is important that students receive regular feedback on their work.

V. List of Major Course Topics

The following list of topics will be covered in the course.

1. Introducing Real Numbers
2. Completeness and consequences of Completeness
3. Neighborhoods, Open and Closed Sets
4. Sequences and Series**
5. Functional Limits and Continuity
6. Uniform Continuity
7. Derivatives
8. Riemann Integral, Fundamental Theorem of Calculus

** as time permits

Approved 4/8/15

MAT 310 Real Analysis

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Instructor: Judit Kardos

All communication outside of class will be handled by e-mail: kardosj@tcnj.edu

Expectations for response times: *24 hours on weekdays, 72 hours on weekends*

Meeting schedule:

Tue-Fri at 3:30pm SCP 230

Studying Real Analysis requires at minimum 8 hours per week, in addition to the class meetings. Please keep this in mind as you set up your responsibilities and schedule. Success in this class depends solely on the amount of work you do inside and outside of class.

Office hours: Tuesday 10-11am (in person), Wednesday 4-5pm (in person or online) Friday 5-6pm (in person)

Textbook: *Steven Abbott, Understanding Analysis,* Springer, 2nd edition

Prerequisites: MAT 200 or (CSC 270 and permission from the chair) and MAT 229

Course Description

In this course you will discover and prove theorems that you first encountered in Calculus. In Calculus you did not prove any of the theorems rigorously. You learned some algorithms for computing derivatives and integrals. The collection of these algorithms is called a ``calculus''. Before an algorithm is used we must know that it leads to correct results. Therefore each algorithm or formula has to be rigorously proved by a mathematician. Your freshman Calculus is more fully described as Differential and Integral Calculus. Using the rules and formulas from Calculus, a scientist or engineer can solve more problems than just using algebra.

So what do mathematicians do? Mathematicians are the ones who go and prove the rules and algorithms that everyone uses. A mathematician can be viewed (although a bit incompletely) as the tool maker.

Mathematicians also do more. They study math for the sake of discovering new math knowledge. They also dig deeper and try to assure that what we know is secure and eternal knowledge. Mathematicians time to time face a crisis where paradoxes indicate that there is a need to make our subject even more rigorous. That is what happened at the end of the 19th century when we needed to address basic questions of Calculus: What is a number? What is a limit? What functions have derivatives? What is the area of an object? When can we define a function with a polynomial of infinite degrees? That is how Real Analysis was born.

As a math major in this course, you will be asked to think like a mathematician. As a mathematician, you will construct proofs from the basic axioms of the real numbers, you will double check what assumptions are necessary in order to prove a result, you will look for counterexamples, you will double-triple check every step in any reasoning that you will create. In short, you will elevate your thinking to a higher level. You may never use the topics of Real Analysis in your life. But learning the method of Real Analysis will change your thinking and your life forever. The clarity and precision of the thought processes used in Real Analysis will serve you the rest of your life. You may think about studying Real Analysis as creating a very complicated magic key that will open any simple door that you will encounter in life.

The process of proving theorems can be organized into two main components. The first component of creating a proof is to solve the problem conceptually. The second component is to demonstrate using formal mathematical arguments that you have solved the problem.

You are strongly encouraged to go slow, and take your time solving problems conceptually. There are no formulas or algorithms in this course. Draw pictures, look at examples, use lots of scratch paper, and think! Most importantly, you should not be afraid to explore and try something even if it is wrong. By analyzing failed attempts, you can often gain greater insight.

Once you understand a problem and you think you've solved it, then turn your attention to writing up and demonstrating your solution. Demonstrating your solution may require more than one attempt before you arrive at your final, finished product. You will have to revise often and write several drafts of proofs.

In this course we begin to explore deeper properties of the real line and real functions. We will work with limit processes by first exploring sequences of real numbers and then applying the sequential limit concept to the notions of continuity and differentiability. At the end of the semester, the definition of the definite integral of a function will be carefully examined as an example of a more general limit process.

Learning Goals

1. Improve your problem-solving, proof writing and presentation skills.
2. Understand why an intuitive understanding of calculus was insufficient as a foundation for building more complicated and powerful mathematical tools in analysis.
3. To enjoy the payoff of analytical rigor and abstraction by studying questions that give analysis its inherent fascination.

4. To acquire a thorough understanding of the concept of a limit in many different settings.
5. Learn to think clearly and rigorously. Learn to divide a problem into smaller parts, understand each part completely and then synthesize the parts into a complete solution of the problem.

Learning Activities

This course will likely be different from other math courses that you have taken.

First, you are not going to be asked in this class to complete tasks like "solve for x" or "integrate this function". Mathematicians are in the business of understanding complex problems deeply and solving them by making connections, reinterpreting questions and finding new ways to look at things. This course will teach you to think like a mathematician. If you try to simply memorize things, you will fail tests and quizzes, since questions will require that you make sense of things and use your knowledge in creative ways. Unfortunately, short term memory shuts down under stress, leading to underperformance. So it is important to minimize memorization when you are studying in order to reduce the effect of your anxiety on your grade. How can you do that?

Instead of relying on memorization, focus on quizzing yourself, finding examples, solving problems independently, and engaging in active discussions with your group members. Break every proof down into smaller steps that you fully understand. These strategies help move information into long-term memory, which, unlike short-term memory, remains accessible even when you're feeling anxious. These approaches, known as "desirable difficulties," have been shown to be effective by neuroscientists. The term was coined by Robert Bjork in 1994. Research has also shown that simply reading, re-reading, and highlighting your textbook or notes is not an effective use of your time.

You will need to actively engage in constructing your own understanding of the material. Many of the concepts and problems you encounter will be completely new and will challenge you to stretch your thinking. You will likely experience frustration and failure before achieving understanding. This is a normal part of learning process mathematics. If you're approaching the material correctly, you should expect to feel confused at various points throughout the semester. The concepts of analysis are too complex to be fully grasped all at once.

Students in this course will be required to:

- read and actively interact with course notes, videos and the textbook on their own
- work on online pre-class quizzes and later correct their answers
- work in teams on the homework assignments
- write up quality solutions and complete proofs to assigned problems
- present solutions on the board to the rest of the class
- participate in discussions centered around a student's presented solution
- call upon your own prodigious mental facilities to respond in flexible, thoughtful, and creative ways to problems that may seem unfamiliar on first glance

Assessment

1. Homework: Hand in homework will be collected regularly. We will form homework groups (3-4 people per group) in class. Everyone will have to submit their own copy of the homework but only a few problems will be graded per person. The group will receive the sum of the grades.

2. Examinations: We will have 4 quizzes, two tests and the Final Exam.

We will have the first quiz, Quiz 0, on Logic. Quiz 0 will have similar problems to the Discrete review quizzes and Homework 0.

4. Participation: **Your participation grade will be determined by your attendance of weekly group meetings, class meetings, tutoring sessions and participation in the discussions and online quizzes.** If there is a special circumstance that prevents you from participating, please email me.

Anyone missing more than three classes or group meetings without excuse will lose 1% of semester grade for each additional missed class or meeting.

Grading Scale:

93-100 A
90-92 A-
88-89 B+
80-87 B
79 B-
77-78 C+
70-76 C
69 C-
67-68 D+
60-66 D

Make-ups

Make-up exams will only be given in extraordinary circumstances, when written documentation of the emergency is provided to me. Details on TCNJ's College Attendance Policy can be found at: http://policies.tcnj.edu/policies/digest.php?docId=9134

Grading

6 Homework sets: 15% (2.5% each)

Online Quizzes: 6%

Discrete Review: 4%

Quiz 1-3: 25%

Test 1: 15%

Test 2: 15%

Final: 20%

4th hour use: Students are required to participate in a weekly Group meeting to satisfy the 4th hour requirement.

Academic integrity: You are expected to know the college's policy on academic integrity, which can be found at http://policies.tcnj.edu/policies/digest.php?docId=9394. While I encourage you to work with your classmates on assignments, each write-up must represent your own work. Further, the use of solution manuals and online tools to do the homework is a form of cheating, although in reality you only hurt yourself by not putting effort into the homework. Cheating on exams in any form will not be tolerated.

Students in need of accommodations:

Students with documented needs for in-class accommodations must submit paperwork by the second week of the semester. All documented accommodations will be respected, as specified by the Americans with Disabilities Act Policy (http://policies.tcnj.edu/policies/digest.php?docId=9206). Further information can be sought through Disability Support Services: http://differingabilities.pages.tcnj.edu.

Recording of Class Sessions

"In accordance with the Remote Classroom Camera/Microphone Use and Recording Policy our class sessions can be audio-visually recorded (both for students in the class to refer back to and for enrolled students who are unable to attend live). Recordings of your audiovisual participation in the class session are considered "directory information" under the College's Family Educational Rights and Privacy Act Policy. Access to the recordings will be password-protected and available to students in the class through Canvas. In general, a recording will not be maintained beyond the end of the following semester (allowing students who earned a grade of incomplete to finish the course) and thereafter will be deleted unless a compelling need exists for retaining it. If you have concerns about being recorded or having the recording accessed by other students in the class, please contact your instructor or the Accessibility Resource Center, who can endeavor to address those concerns and ensure that the requirements of the Family Educational Rights and Privacy Act and TCNJ Family Educational Rights and Privacy Act Policy and Remote Classroom Camera/Microphone Use and Recording Policy are met."

Accessibility and Accommodations

Students who experience barriers in this course are encouraged to contact the instructor as early in the semester as possible. The Accessibility Resource Center (ARC) is available to facilitate the removal of barriers and to ensure reasonable accommodations. For more information about ARC, please visit: https://arc.tcnj.edu/.

Access to IT support

If you have technology issues or needs during the semester, please contact the IT Helpdesk at 609-771-2660 or helpdesk@tcnj.edu.