

Program Cover Sheet --- MAT 351: Geometry

I. Course Information

MAT 351: Geometry is normally a junior/senior level course. It meets for two 80-minute lecture periods. The prerequisites are MAT 200 and either MAT 205 or MAT 229. The MAT 200 requirement can be met with CSC 270 and permission of the chair.

Geometry, derived from the Greek for “measurement of the earth,” is the study of shapes and their measurements. The ancient Greeks devoted much study to the geometry of the flat plane – an abstract generalization of a farmer’s field. The theorems of the ancient Greeks were collected by Euclid in his 13 volume work *The Elements*, where he developed many of the theorems of what we now know as Euclidean geometry. Most of what is learned in a modern day high school geometry class can be traced back to the ancient Greeks. It was only in the mid-nineteenth century that mathematicians realized that other, equally valid two-dimensional geometries existed. This course will further develop students' knowledge of the geometry of Euclid and expose students to non-Euclidean geometry.

II. Learning Goals

Geometry is a course designed primarily for mathematics education majors. Therefore, one of the fundamental goals of the course is to bring students to a level of familiarity with geometric concepts that is more than sufficient for teaching high school geometry courses. This course is also an excellent course for liberal arts mathematics majors who wish to prepare for further study in graduate school or who simply wish to be exposed to an area of mathematics not addressed in the introductory courses in our department.

In this course, students will be exposed to both Euclidean and non-Euclidean geometries from a variety of mathematical perspectives. In addition to helping students understand the specific geometries involved, this will call upon students to think about a single mathematical object in a variety of ways, both abstract and concrete. Thus, in the process of becoming a competent undergraduate geometer, the student will continue to develop as a mathematician.

This course will entail a highly abstract treatment of those geometric objects with which students consider themselves familiar. This sense of familiarity will be both a help and a hindrance for students. On the one hand, students will be able to build upon concepts with which they are comfortable as opposed to learning an entirely new domain of discourse. On the other hand, the Euclidean Geometry that students learn in high school is only one of the several geometries, each with its own assumptions and metrics that will be covered in this course. Therefore, students will need to learn the limits of the applicability of their previous exposure to geometry.

Students will grow in their ability to reason abstractly, to read mathematics and write proofs, and to find a mature mathematician’s balance between the abstract and the concrete. Drawing upon geometry learned in high school as well as analytic techniques learned in calculus and introducing and applying the algebraic notion of a group, the course will help students learn to integrate bits of mathematics learned in various settings over the years.

III. Assessment

Students will be expected to demonstrate both technical and conceptual competence in geometry. Exams and homework assignments will directly address the key performance goals of the course as listed in the course syllabus guide. These performance goals require students to think abstractly, to apply abstract concepts in concrete settings, and to utilize knowledge gained in previous courses. Students will be expected to write proofs throughout the course, both in homework assignments and on exams. Students will receive written feedback on their homework and exams, which will help the students continue to develop as mathematicians.

IV. Learning Activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance at lectures offered by the instructor and other students, written homework assignments, oral presentations, participation in classroom discussions, and group or individual projects. These activities will address most of the general goals of the liberal arts mathematics program as well as non-professional goals of the mathematics education program. In particular, students will continue learning to read, write, and explain mathematics, to think logically and abstractly, and to apply knowledge learned in previous courses to solve problems in this course.

Departmental Course Syllabus -- MAT 351: Geometry

In this course, we will explore Euclidean, hyperbolic and elliptic geometries from a variety of viewpoints, both ancient and modern. Students will become well-versed in geometry at the undergraduate level. This course will prepare the liberal arts student for graduate study in geometry and topology, and it will provide mathematics education students with a more-than-sufficient background for teaching a course in high school geometry.

The prerequisites for the course are MAT 200 and either MAT 205 or MAT 229. The MAT 200 requirement can be met with CSC 270 and permission of the chair. The notion of a line integral and computations of area involving double integrals will arise. Students will be expected to read mathematics well and to write proofs competently. The course will also draw on notions learned in high school geometry.

Learning Goals

Content

Students in this course will gain acquaintance with both Euclidean and non-Euclidean geometry. Each geometry will be understood from three different perspectives: first, as the collection of theorems following from a particular set of axioms; second, as the two-dimensional geometry arising from a particular metric; and third, as the geometry obtained from a set together by specifying the group of rigid motions on the set. At least one of these viewpoints will be developed in detail.

Many students will enter the course with the misconception that all geometry is Euclidean and will mistakenly assume Euclidean theorems in a non-Euclidean setting. Students will learn which familiar geometry theorems hold in the non-Euclidean geometries as well as Euclidean geometry and which are specific to Euclidean geometry. Students will come to understand that these geometries are equally valid yet different, and that each is a useful tool in an appropriate setting.

The student will become facile in working with the geometric objects in a two-dimensional geometry, including lines, polygons, and circles. The notions of congruence and parallelism will be explored in depth.

Performance

The successful geometry student will be able to do all of the following:

1. Characterize Euclidean and non-Euclidean geometries through
 - a. an axiomatic description
 - b. a description of the appropriate metric
 - c. a description of the rigid motions of the geometry.
2. Recognize a geometry given any of the above characterizations.
3. Compute lengths and areas using a metric.
4. Demonstrate proficiency with rigid motions.
5. Determine congruence or non-congruence of different objects in a particular geometry.
6. Categorize lines in a particular geometry according to their parallelism properties.
7. Prove or disprove theorems in the various geometries.
8. Identify which familiar theorems from Euclidean geometry hold in a particular non-Euclidean setting.

Assessment

Students will be assessed through some combination of graded homework assignments, oral communication, group and individual projects, and in-class and final examinations. The homework assignments and examinations will directly assess the eight performance goals listed above. Formal and informal oral communication provides students with an alternative route for demonstrating competence with one or more of the performance goals and for developing geometric intuition interactively with fellow students and the instructor.

Homework assignments will provide students with opportunities to attempt lengthier, more challenging problems than is possible on an examination as well as offering students practice at examstyle problems. Examinations, which normally preclude both the use of books and the practice of group discussion, enable the professor to assess the knowledge an individual student has readily available.

Learning Activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance at lectures offered by the instructor and other students, written homework assignments, oral presentations, participation in classroom discussions, and group or individual projects.

The course outline will vary according to the preferences of the instructor. However, the outline must address the learning goals given above. Moreover, elliptic geometry should not be the primary focus of the course, and Euclidean rigid motions should be given a thorough treatment. A sample syllabus is attached.

Updated 2/12/2025.

Geometry Spring 2025 Syllabus

Course: MAT 351

Prerequisites: MAT 200 and either MAT 205 or MAT 229. The MAT 200 requirement can be met with CSC 270 and permission of the chair.

Class meetings: Tuesdays and Fridays, 2:00-3:20, SCP 201

Instructor: Andrew Clifford

Office: Science Complex, P242

Phone: 771-3060

Office Hours: Mondays, 11:30-1:00 and Wednesdays 1:30-3:00
other times by appointment.

Text: Saul Stahl, The Poincare Half-Plane, Jones and Bartlett Publishers. It is in its second edition. If you have the first edition, that will be fine.

Outline: This course will augment student understanding of Euclidean geometry by analyzing its historical and axiomatic development. This part of the course is designed to give future mathematics teachers a deep foundation for the standard high school geometry curriculum. After this, we will explore the more modern analytic and algebraic approaches to Euclidean geometries. These will then be applied to develop hyperbolic geometry.

Course objectives: The principal objective of this course is the development of an understanding of and an appreciation for the Euclidean and non-Euclidean geometries from the logical, analytic and algebraic perspectives. Beyond this, we will further develop the ability for reading mathematics, a skill which will be useful in future math courses and essential for those who make a career of mathematics or its instruction. For this reason, students will be responsible for regular reading including some topics that will not be discussed in class.

Learning activities: Learning activities will include of a combination of: in-class lectures; readings; and group work, both in and out of the classroom.

Group quizzes: Several times this semester, we will devote class time to group quizzes. The process for these will be as follows: I will give the class descriptions of the problems that will be on the quiz. Students will be allowed to look at course resources and discuss the questions with each other for 5 minutes. Then, I will give the class the quiz questions at which time students will have put away resources and ceased conversation.

These quizzes will be unannounced. There will be no make-ups. I will drop the lowest quiz grade.

Tests: There will be two in-class tests this semester, March 7 and April 18. There will be a cumulative final exam during finals week. Make-up tests will be given only in extreme circumstances and by prior arrangement. Please let me know as soon as possible if you will not be able to take a test when it is scheduled.

Grading:

In-class tests, 3/7 and 4/18: 40%

Quizzes: 20%

Class participation: 10%

Final exam, TBA: 30%

I reserve the right to adjust grades according to the more nebulous facets of student performance. [For example, insightful contributions to class discussion will tend to improve your grade while poor attendance will have the opposite effect.]

Academic Honesty: Please familiarize yourself with TCNJ's Academic Integrity Policy. It is available here: <http://www.tcnj.edu/~academic/policy/integrity.html>. I will confront any suspected violations in strict accordance with this policy. I am happy to answer any questions about the general policy or about the rules which are specific to our class. You are responsible for knowing these rules and the consequences of breaking them.

Other useful TCNJ Policies: Other academic policies can be found here:

<https://academicaffairs.tcnj.edu/tcnj-syllabus-resources/>

This syllabus is subject to change. I will announce such changes in class.