**Program Cover Document --- MAT 326: Differential Equations**

## I. Basic Course Information

MAT 326: Differential Equations is a 1 course unit course. It will have two 80-minute meeting periods each week. MAT 326 is primarily a sophomore/junior-level course. The prerequisite is MAT 128 (Calculus B). Another prerequisite is one of MAT 200 (Proof Writing through Discrete Mathematics), MAT 205 (Linear Algebra), and MAT 229 (Calculus C). The MAT 200 requirement can be met with CSC 270 and permission of the chair. It is required of all students in the Applied Mathematics specialization, and is an option for all other specializations in the Department of Mathematics & Statistics.

*Prerequisite*: MAT 128 and (MAT 200 or MAT 205 or MAT 229). The MAT 200 requirement can be met with CSC 270 and permission of the chair.

Course Description (For Bulletin): The subject of Differential Equations is the study of models of nature. Historically developed principally for the study of problems in physics, now differential equations have a much broader application to chemistry, biology and the social sciences. Differential Equations is one of the foundational topics in the modern day study of dynamical systems. Students will draw upon their knowledge of calculus to solve many types of differential equations using a variety of analytical, qualitative, and numerical techniques.

## II. Learning Goals

Students will acquire proficiency in solving a wide variety of differential equations using many different methods and approaches. Students will develop an appreciation and understanding of how mathematical models describe real-world phenomena from biology, chemistry, physics, and/or economics. Mathematical proofs often help students appreciate the subjects at an elevated level and will be shown in class and given as written assignments throughout the course. Technology is a powerful aid in understanding the behavior of differential equation solutions, particularly when an analytical solution cannot be attained.

At the end of this course, students will be able to

- Understand the mathematical underpinnings of differential equations and their solutions. This will include classification and comparison of differential equations, criteria for the existence/uniqueness of solutions, and the calculation of general solutions.
- Develop an intuitive feeling for the approach, creation and formulation of a mathematical model that addresses a problem in nature. In particular, students will understand the importance of mathematical applications to science and how solving differential equations can answer real-world problems.

- Obtain and describe solutions analytically, qualitatively, and numerically, using software packages to visualize and interpret the behavior of solutions.
- Develop mathematical sophistication in their ability to solve complex problems with multiple steps and requiring several techniques (ranging from analytical, to qualitative, to technological).

## III. Learning Activities

Learning activities will consist of combinations of interactive lectures, group work, discussion, presentations, and/or computer programming assignments. Individual instructors will tailor their activities according to their goals and preferences. Outside of class, students are expected to do a significant amount of individual and in some cases group work to achieve learning goals. By giving students a variety of approaches to learning and doing mathematics, these activities should promote a deeper understanding of those ways that differential equations and modeling in general help to ameliorate problems encountered in nature. Students will receive detailed feedback on their writing assignments. The course will include multiple substantial examinations and/or projects, on which students will receive additional detailed feedback.

## IV. Student Assessment

Students will receive regular feedback on their work through written assignments, quizzes, projects and/or examinations. Certain assignments may be assigned via an online homework system that provides instant feedback to students and assesses student progress and understanding of the course material.

## V. List of Majors Course Topics Course Outline and Required Course Topics

The required mathematical topics for the course, which should be supplemented with the appropriate use of technology are listed below.

* Indicates the optional topics to be covered if time permits.

1. **Introduction and Basic Differential Equation Terminology**

2. **First Order Differential Equations**
    a. Existence and uniqueness of solutions
    b. Direction fields
    c. Equilibria and stability analysis
    d. Separable differential equations
    e. Linear differential equations
    f. Principle of superposition

g. Substitutions and transformations; change of variables
h. Euler's Method
i. Improved Euler's Method*
j. Exact Equations*
k. Bifurcation analysis*

**3. Mathematical Modeling and Compartmental Analysis**

a. Mixing Problems
b. Population Models
c. Mass-Spring Models*

**4. Second Order Differential Equations**

a. Linear homogeneous differential equations with constant coefficients
b. Linear nonhomogeneous differential equations and the method of undetermined coefficients
c. Linear nonhomogeneous differential equations and variation of parameters
d. Linear homogeneous differential equations with variable coefficients, and the Reduction of order method and Cauchy-Euler equations e. The Laplace transform

**5. Series Solutions of Differential Equations**

a. Power Series Solutions of first order and second order equations near ordinary points*
b. Regular singular points and series solutions near regular singular points*

**6. Systems of Differential Equations**

a. Linear systems – finding solutions using the elimination method*
b. Linear systems – finding solutions using eigenvalues and eigenvectors
c. Phase plane and classification of equilibria for systems of equations
d. Nonhomogeneous linear systems of differential equations*
e. Nonlinear systems of differential equations and linearization*
f. Numerical methods for systems of differential equations*

*Approved 5/6/2024*

# DIFFERENTIAL EQUATIONS
# MAT 326-01, Spring 2024

## CATALOG DESCRIPTION

*MAT 326/Differential Equations*: This course is an introduction to differential equations and in particular their applications to real world phenomena. Topics include: first order ordinary differential equations and analytical/numerical/qualitative approaches to studying their solutions, mathematical modeling and compartmental analysis, homogeneous and nonhomogeneous linear differential equations, series solutions to differential equations, and an introduction to systems of linear differential equations. Current technology such as MATLAB and Mathematica will be used to explore solution features. (1 course unit)
*Prerequisite:* MAT 128 and (MAT 200 or MAT 205 or MAT 229). The MAT 200 requirement can be met with CSC 270 and permission of the chair.

## INSTRUCTOR INFORMATION

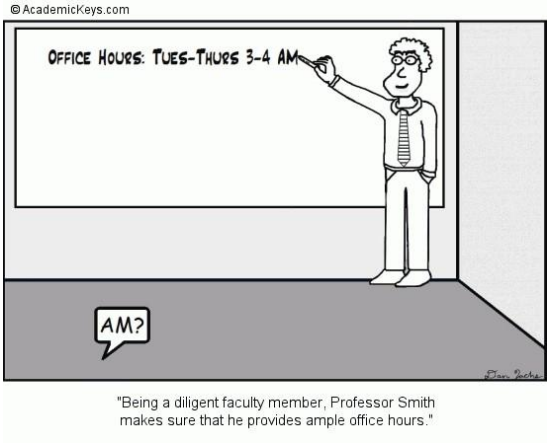
**Instructor**: Dr. Jana Gevertz

**Office**: Physics & Mathematics P246

**Email**: gevertz@tcnj.edu **Phone**: 609-771-3314

**Office (Student) Hours**: M 4-5:30, We 4:15-5:45, Th 10-10:45

I really hope to see many of you at office hours, whether you have questions or just want to chat! Be aware that office hours may occasionally be held over Zoom, rather than in person.



"Being a diligent faculty member, Professor Smith makes sure that he provides ample office hours."

## COURSE INFORMATION

**Time**: Monday and Thursday, 2:00-3:20

**Location**: Physics & Mathematics P228*

*On occasion, class may be held over Zoom. On those days, you are still welcome to attend from our physical classroom, or you can attend from the comfort of any space you'd like.

**Course Website**: This course utilizes the Canvas course management system. General course material, pre-class readings, online quizzes, Zoom links, homework assignments, and all other important documents will be posted on Canvas.

**4th Hour**: In this class, deep learning outcomes associated with TCNJ's 4th hour are accomplished by a series of rigorous educational assignments that extend beyond the typical scheduled class time. These include out-of-class tutorials and assignments to develop proficiency with the MATLAB software package.

## COURSE MATERIALS

**Required Text**: *Differential Equations with Boundary-Value Problems* by Zill, 10th Ed.

**Required Software/Technology**: The course will use the mathematical software package MATLAB. It is freely available for download onto your personal computer at the following link: https://software.tcnj.edu/. *I recommend you try and download it now!!* If you prefer, you can also access MATLAB through a web interface, without having to download anything, at either TCNJ's app page or through MathWork's TCNJ access site.

The course may also occasionally use Zoom video for live class and office hours. You must be logged on to your TCNJ account to enter the Zoom classroom. If you haven't activated your TCNJ Zoom account yet, you can do so with TCNJ's Single Sign On. Having a computer/laptop/tablet with a microphone and video capabilities will greatly facilitate your participation in the course. If you do not have access to the necessary equipment for this course, TCNJ can provide it for you for the semester. Please contact care@tcnj.edu to help you secure the needed technology free of charge. This is a confidential process. However, if you need assistance in this process, please let me know. Also, the Accessibility Resource Center (ARC) is available to help facilitate the removal of barriers and to ensure reasonable accommodations if needed.

**Recommendation for Note Taking*:*** I highly recommend you use a three-ring binder instead of a notebook for the course, as handouts will be regularly distributed to guide us through pre-class readings, lectures, and group work. Alternatively, all handouts will be posted on Canvas as PDF files, so you can also work on a tablet if you prefer.

## COURSE PURPOSE & LEARNING GOALS

Every living and many nonliving things are in a constant state of change. For instance, cancer can grow within an organism, a pollutant can spread through a lake, and oil prices can rise and fall. Differential equations are used to model the *rate of change* of these, and other natural phenomena. Solutions of the differential equations that model these processes allow one to make predictions on future behavior. For example, we can use the solution to the appropriate differential equation(s) to predict how a cancer will grow, to quantify the extent to which a lake is polluted, and to predict the cost of oil at a particular point in the future.

Many differential equations that arise out of real-world phenomenon cannot be solved analytically, and instead require the use of qualitative and numerical solution methods. Therefore, in this course, we will focus on why we study differential equations, and we will learn how to understand the behavior of these equations using analytical, qualitative, and numerical techniques. The learning goals of this class are to:

1. Create and analyze mathematical models using ordinary differential equations.
2. Classify any given differential equation, determine if a solution exists, and select the appropriate analytical technique (if there is one) for finding the solution.
3. Utilize qualitative methods to gain insight into the behavior of a single differential equation and systems of differential equations.
4. Exploit technology, particularly MATLAB, to find analytical, graphical, and numerical solutions of differential equations.

## COURSE POLICIES


**Class goals:** Our classroom is intended to be an equitable and inclusive learning environment. If something about the course is not working for you, you can reach out to me so we can fix that. If you prefer to leave anonymous feedback, you can do so at: https://forms.gle/574ezrHLwjAfj6ZL6.

**Class structure:** Class will be a mix of interactive lecture and group work. It is highly encouraged that you actively participate in class and treat it as a comfortable environment in which to ask questions, answer questions, and make mistakes. Before most classes, you will have a brief required reading (2-4 pages) to prepare you for the upcoming class. You will be given a quiz, with 3 attempts and automated feedback, after completing the reading. This goal of the pre-class readings is to give you a gentle introduction to what will be covered in class, freeing up time for interaction and active learning during class.


**Assessment**: The following formula will be used to calculate your final grade. My intention is that this grading system offers you the opportunity to learn through practice and mistakes. One disappointing exam, or one missed assignment, will not doom your grade.

| | |
|---|---|
| **Homework**<br>Anticipate weekly assignments consisting of WebAssign problems, MATLAB problems, and hand-written problems to be submitted. Lowest grade will be dropped. | 20% |
| **Asynchronous Participation**<br>Pre-class contributions on Perusall (must contribute to 50% of readings), pre-class quizzes on reading, group work Canvas quizzes, post- class reflections (if implemented) | 15% |
| **Exam #1**<br>Tentative Date: Thursday 2/22 | 5% or 20%<br>(5% if its your lowest exam) |
| **Exam #2**<br>Tentative Date: Thursday 4/4 | 5% or 20%<br>(5% if its your lowest exam) |
| **Exam #3**<br>Tentative Date: Thursday 5/2 | 5% or 20%<br>(5% if its your lowest exam) |
| **Final Exam**<br>Date TBA | 20% |

In addition, if your average ends up on the border of two letter grades, very good participation can bump up your final grade by one-third a letter grade. *The instructor reserves the right to change or modify this formula as needed.*

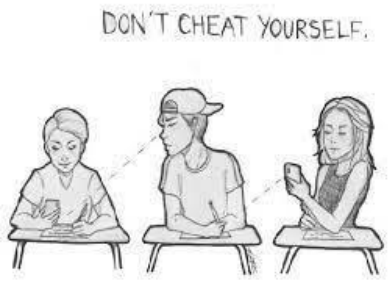
**Homework, Quizzes, and Group Work**: Before each class, you will have a brief required reading to prepare you for the upcoming class. After completing the reading, you will be given a Canvas quiz that allows you up to 3 attempts to get all answers correct. The other component of your quiz grade will be your answers to your group work problems.

Web-based homework will be assigned to you almost every class day via WebAssign. These homework assignments will be graded instantly upon submission (you get 10 attempts per problem, unless it is a multiple-choice problem), and will be the one of the main determinants of your homework grade. Although assignments are submitted online, you should complete each problem in a homework notebook. In this notebook you should clearly explain how you approached each problem. WebAssign assignments will be due by 11:59pm on the due date. You can set up your WebAssign account using the directions given here: https://startstrong.cengage.com/webassigncanvas-ia-no/. *I recommend you set up your account now!!*

You will also be assigned written homework that assesses the more theoretical course topics. These assignments will be announced in class and posted on Canvas. You are responsible for knowing all due dates whether you are in class the day an assignment is given or not. You are encouraged to work on the homework with your classmates, although your final write-up must be your own.

**Attendance and Late Policy**: Homework, quizzes, and exams are based on material presented in class, so attendance during class is integral to learning the course material. While it is important to try and stay up to date on the course work, I understand that life happens. Please don't hesitate to come to me if circumstances are not permitting you to meet a deadline, or if you are anticipating an absence for a religious observance or for attendance at an official College event. That said, I ask that you come to me *before an assignment is due or a class is missed* (whenever possible) if you find you need additional time to complete an assignment or if you will not be attending class. The policy around exams is stricter. Makeup exams will only be given if circumstances truly warrant one. Details on TCNJs College Attendance Policy can be found here.

**Academic integrity**: You are expected to know the college's policy on academic integrity, which can be found here. While I encourage you to work with your classmates on assignments, each write-up must represent your own work. Please refrain from representing the work of others, *including solutions/code obtained online or through AI*. If you use an online resource to support your solutions/code, simply include a citation and indicate how you used the reference. If you gave a prompt to an AI system, include the prompt and the output. If something about the way the course is functioning is tempting you to violate our academic integrity policy, then it is likely there is more I can do to support your learning. In that case, please let me know what I can do to make the class work better for you!



https://thewolfpacket.org/2604/news/how-strict-is-the-chs-academic-policy-is-it-really-upheld/

**Students in need of accommodation**: Students who experience barriers in this course are

encouraged to contact the instructor as early in the semester as possible. The Accessibility Resource Center (ARC) is available to facilitate the removal of barriers and to ensure reasonable accommodations. All documented accommodations will be respected, as specified by the Americans with Disabilities Act Policy.

**Background**: Success in Differential Equations depends on your knowledge of single variable calculus. While we will do just-in-time activities to jog your memory on some needed topics, you may also want to review them on your own:

- *Definition of the Derivative* (Stewart: §2.7, 2.8)
- *Differentiation Rules* (Stewart: §3.1-3.4)
- *Implicit Differentiation* (Stewart: §3.5)
- *Linear Approximations* (Stewart §3.10)
- *Techniques of Integration* (Stewart: §5.5, 7.1-7.4)
- *Power Series* (Stewart: §11.8)
- *Taylor Series* (Stewart: §11.10)

## INTENDED TOPIC AND ASSIGNMENT SCHEDULE

** Note that the contents of this syllabus are subject to change at the Professor's discretion **

| Class Date | Textbook Section(s) | Topic(s) | Assignments Due |
|---|---|---|---|
| M 1/22 | 1.1a | 1. Welcome; Intro to DEs | |
| Th 1/25 | 1.1b, 1.2a | 2. DE Solutions and IVPs | HW #1 |
| M 1/29 | 1.2b | 3. Existence & Uniqueness | |
| Th 2/1 | 1.3, 3.1, 3.2 | 4. DEs as Mathematical Models | HW #2 |
| M 2/5 | 2.2 | 5. Separable Equations | |
| Th 2/8 | 2.3 | 6. Linear Equations | HW #3 |
| M 2/12 | 2.5 | 7. Solutions by Substitution | |
| M 2/19 | | Catch up<br>8. Choosing Solution Strategy (pre-reading only) | HW #4 |
| Th 2/22 | | **Exam 1** (though Substitutions) | |
| M 2/26 | 2.1a | 9. Phase Portraits | |
| Th 2/29 | 2.1b | 10. Direction Fields | HW #5 |
| M 3/4 | 2.6 | 11. Euler's Method | |
| Th 3/7 | 1.3, 3.1, 3.2 | 12. Linear DE Models | HW #6 |
| M 3/11 and Th 3/14 | | Spring Break! | |
| M 3/18 | 4.1 | 13. Theory of Linear Equations | |
| Th 3/21 | 4.1 | 13. Theory of Linear Equations | HW #7 due |
| M 3/25 | 4.3 | 14. Homogeneous Linear Equations with Constant Coefficients (Real Roots) | *[Note: 3/25 is last day to withdraw]* |

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|---|---|---|---|
| Th 3/28 | 4.3 | 15. Homogeneous Linear Equations with Constant Coefficients (Complex Roots) | HW #8 due |
| M 4/1 | 4.4 | 16. Undetermined Coefficients | HW #9 due |
| Th 4/4 | | **Exam 2** (through Theory of Linear Equations) | |
| M 4/8 | 4.4, 4.6 | 16. Undetermined Coefficients;<br>17. Variation of Parameters | |
| **Class Date** | **Textbook Section(s)** | **Topic(s)** | **Assignments Due** |
| Th 4/11 | 4.6, 4.7 | 17. Variation of Parameter;<br>18. Cauchy-Euler Equations | HW #10 due |
| M 4/15 | 6.1 | 19. Review of Power Series | |
| Th 4/18 | 6.2 | 20. Solutions about Ordinary Points | HW #11 due |
| M 4/22 | 6.2 | 20. Solutions about Ordinary Points | |
| Th 4/25 | 3.3, 8.1 | 21. Systems of Linear DEs | HW #12 due |
| M 4/29 | 8.2 | 21. Systems of Linear DEs | |
| Th 5/2 | | **Exam 3** (though Power Series Solutions) | HW #13 due 5/6 |