Program Cover Document - STA 404: Computational and Bayesian Statistics

I. Basic Course Information

Computational and Bayesian Statistics is primarily a junior/senior level course. The course is designed to introduce students to modern computational methods of parameter estimation in multidimensional spaces and to Bayesian methods.

Course prerequisite: STA215 or STA 216 or (ECO 105 and (MAT 125 or MAT 127)) or (STA 145 and MAT 127)
MAT 316
CSC 120 or MAT 203 or CSC 220

The study of statistics has been greatly impacted by the development of computational power over the last 20 years. The course will focus on two main areas. First, several key methods developed in classical statistics that rely on computational power will be introduced. Second, computational Bayesian statistical methods will be introduced. In both cases, students will learn the methods by solving problems from the point of view of classical statistics (confidence intervals, classification) and within scientific studies (spectral analysis, imaging, model selection).

The development of modern computing has led to the ability to utilize the previously intractable approach introduced by Bayes and Laplace and updated by Jaynes in the 20^{th} century. The use of these methods, however, requires the combination of the concepts of conditional and marginal probability of distributions learned in MAT316 and substantial programming skills gathered in introductory programming courses and refined to the application in statistics or mathematics in 300-level courses.

II. Course Description

This course will present an introduction to computational and Bayesian statistics. Topics include Bayesian computational methods, including the effect of prior knowledge in estimation and setting of confidence intervals, and the impact of computational methods on non-Bayesian methods. Substantial statistical programming will be involved.

III. Learning Goals

This course will expose students to several computational and Bayesian statistical methods. By the end of the course, the students will be able to:

1. Convert a physical model or observational data set to a computational statistical model.
2. Estimate sampling and null distributions for any univariate statistic using bootstrap and permutation methods.
3. Utilize techniques developed in the course to estimate parameters in low and high

dimensional parameter spaces.

4. Perform variable transformations to convert a physical model to a statistical model for the observations generated.
5. Develop and code Markov chain Monte Carlo estimation methods for high dimensional parameter spaces.
6. Understand and derive Cox's rules of inference and distinguish Bayesian and Laplacian methods of inference from those of Fisher and Pearson.

Given that this is a 400-level course, students will improve their ability to work independently on multi-step problems.

IV. Learning Activities

Learning activities may consist of a combination of lectures, group work, and significant review of programs. The specific choice will depend on the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals, including creation of complex code for parameter estimation and visualization.

V. Student Assessment

Students will receive feedback on their work through either homework assignments, projects, and/or examinations.

VI. List of Major Course Topics

The following list of topics will be covered in the course. Items in the right column are optional; however it is expected that the instructor will choose at least two of these topics for inclusion in the course. Each topic involves both theoretical derivation and computational instantiation of the methods.

Required topics	**Optional topics**
1. Estimation of Sampling Distributions	
a. Classical statistical examples and their limits	
b. The Bootstrap	
c. Empirical null distributions	
2. Expectation-Maximization	
a. Two-Class estimation	
b. Sensitivity and Robustness in EM	
	c. Bayesian EM
3. Rules of Inference	
a. Cox's rules of inference	

b. Bayesian vs Frequentist concepts of inference

4. Bayes' Equation
    a. Joint Probability to Bayes' Equation
    b. Interpretation of Prior Probabilities
    c. The Log-Posterior Space and the Evidence
    d. Conjugate priors
    e. Philosophical motivation

5. Point Estimation in One and Two Dimensions
    a. Maximization of the Log-Posterior
    b. The Effects of Priors
    c. Taylor Series Approximation of the Log-Posterior
    d. Newton-Raphson method

6. Posterior Estimation
    a. Absolute Probability and Posterior Shape
    b. Monte Carlo Estimation
    c. Acceptance-Rejection Sampling
    d. Sampling Importance Resampling
    e. Integration with conjugates

7. Markov Chain Monte Carlo
    a. Discrete Markov Chains
    b. Steady States and Classification of State
    c. The Ergodic Theorem
    d. The Metropolis Algorithm
    e. The Metropolis-Hastings Algorithm
    f. Continuous Markov Chains
    g. Gibbs Sampling
    h. Variational Bayes
    i. Maximum Entropy
    j. Massive Inference
    k. Nested Sampling
    l. Reverse-Jump MCMC

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