

Program Cover sheet - MAT 105: Mathematical Structures and Algorithms for Educators I

I. Basic Course Information

Purpose statement: In 2012, the Conference Board of Mathematical Sciences, in conjunction with the American Mathematical Society and the Mathematical Association of America, the nation's two primary national mathematical organizations, published a report entitled, *The Mathematical Education of Teachers II*. Among its recommendations are:

- Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.
- Prospective elementary grade teachers should be required to take at least 12 semester-hours on fundamental ideas of elementary school mathematics.

MAT 105 is part of an experience that is aimed at meeting these recommendations.

It is especially appropriate for Inclusive Education majors. MAT 105 will study the fundamental principles that underlie elementary school mathematics from an advanced viewpoint.

II. Learning Goals

In MAT 105, students will engage in a thorough development of fundamental concepts, such as patterns, numeration, algorithms, mathematical systems, real numbers, and quantitative reasoning. Through this process, they will develop understanding of the processes and algorithms found in elementary mathematics and discover purposes beneath the symbols and techniques. In addition, upon completion of this course, we expect students to show an improved ability to communicate mathematical ideas appropriately using the language of mathematics, to reason mathematically, to solve various types of problems using appropriate strategies, and to relate mathematics to other subjects, its applications in society, and to other mathematical topics.

The profound understanding of fundamental mathematics that students gain in MAT 105 will prepare them for the methods course that they take in their sophomore year, where they learn methods and strategies for teaching elementary mathematics to various types of learners.

III. Student Assessment

Students will receive regular feedback on their work through the assignment of homework, written and oral communication, group and/or individual projects and/or explorations, and examinations. Through this feedback, students will be able to see and correct their misunderstandings and improve their performance. Student performance on these assessment instruments and the performance of students in their future mathematics methods courses and field placements will be used to assess the success of MAT 105 in achieving its learning goals and its contribution to the fulfillment of the Inclusive Education program goals.

IV. Learning Activities

In the mathematics methods course that students take in the junior year, they will learn a variety of teaching strategies appropriate for teaching elementary mathematics. They will learn about New Jersey state and national standards for K-6 mathematics, and how to implement these standards in the classroom.

Students must experience standards-based teaching and learning in order to understand how to implement it. Learning activities will consist of a combination of lectures, explorations, group work, participation in class discussions, readings, written homework assignments, and group or individual projects. It is important that a variety of strategies and methods of instruction be used to model effective teaching of mathematics.

Sample syllabus (Specific details can vary, but all of the sections below must be included.)

MAT 105 Mathematical Algorithms and Structures for Educators I
PROFESSOR
OFFICE ADDRESS
EMAIL ADDRESS
OFFICE HOURS
PRIMARY MEANS OF COMMUNICATION
EXPECTED RESPONSE TIME

Course Description and Purpose:

Why do we “invert and multiply” when dividing fractions? How do other numeration systems compare to our base ten system? How do we know that $0.9999\dots = 1$? How can we visually represent multiplication of any type of numbers? Is 0 odd or even? Why can’t we divide by 0? These are the types of questions we will answer in this class.

This course concerns a thorough development of fundamental concepts, such as patterns, numeration, algorithms, mathematical systems, real numbers, and quantitative reasoning. We will study the fundamental principles that underlie elementary school mathematics from an advanced viewpoint, building on knowledge that you bring with you from your K-12 education.

You will be required to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality, using a variety of representations of mathematical concepts and procedures. Physical materials and models will be used to explore fundamental properties of number systems, to model algorithms, and to explore selected algebraic structures. This course is especially appropriate for Inclusive Education majors.

Course prerequisites:

This course will study the fundamental principles that underlie elementary school mathematics from an advanced viewpoint, building on knowledge that students bring with them from their K-12 education. Students must be in the Inclusive Education major to enroll or have the permission of the instructor.

Course Materials:

- Textbook: *Mathematics for Elementary School Teachers*, 7th edition by Bassarear & Moss. We will cover Chapters 1-4 and possibly Chapter 6 in this course.
- Canvas: Explorations Manual, Assignments, materials, and grades will be posted here.

Learning Goals

Content goals: In MAT 105, students will engage in a thorough development of fundamental concepts, such as patterns, numeration, algorithms, mathematical systems, real numbers, and quantitative reasoning. Through this process, you will develop understanding of the processes and algorithms found in elementary mathematics and discover purposes beneath the symbols and techniques. You may enter the course with the misconception that mathematics is all about computation and following procedures. Through careful consideration of many types of

problems and alternative algorithms, you will come to understand that mathematics is much more than computation, and that there is often more than one approach to solving any problem.

Performance goals: By the completion of the course, the successful student will be able to demonstrate all of the following:

- Understanding of the processes and algorithms, and the purposes beneath them, found in the elementary mathematics topics mentioned above.
- An improved ability to communicate mathematical ideas appropriately using the language of mathematics.
- An improved ability to reason mathematically.
- A willingness and ability to solve various types of mathematical problems using appropriate strategies.
- Knowledge of the relationship of mathematics to other subjects, its applications in society, and relationships within mathematics itself.
- An appreciation of the history, structure, and application of mathematics.
- Apply formal logic or numerical analysis to evaluate a quantitative claim, guide a decision, solve a problem, or answer a question.

Format:

The course you are about to begin may be different from any math course you've had before. In this course you will learn the mathematics you need to become a more effective teacher. This is not a methods course. You will take MTT 202 to learn methods of how to teach mathematics. However, it is often said that teachers teach in the manner in which they learned a subject.

What kind of mathematics do teachers need to know? It is mathematics that helps teachers better understand how their students are thinking about mathematics and how they can help their students improve their thinking. It is mathematics that helps teachers see how the different topics in elementary and middle school fit together. It is mathematics that helps teachers see behind all the things they have learned before so they can understand why they work and so they can help their students understand.

This is a different kind of mathematical knowledge. Learning this mathematics requires that you start fresh, that you 'unpack' your current and previous mathematical understandings. It requires that you become curious, genuinely curious, about how and why all those rules in mathematics work like they do, and about the methods children are likely to use to solve mathematical problems and why some of these methods are useful for them and some are not.

Because you will be thinking about elementary topics in mathematics in a new way, you will notice very soon that you are not doing the usual kinds of things. You will not be shown how to do some sample problems and then asked to practice more on your own. Instead:

- You will solve new kinds of problems that are designed to provide insight into several mathematical topics and promote your mathematical reasoning skills. Thus, you will understand familiar problems more deeply.

- You will examine patterns and structure; formulate generalizations and conjectures; investigate and test your conjectures using concrete or virtual materials, calculators, and other tools; and construct and evaluate mathematical arguments. You will learn to ask yourself: Is there a pattern? What might happen next? Can I make a generalization or conjecture? Do I think my conjecture is true for all cases, true for some cases, true for no cases (that is, false for all cases), and why? Under what circumstances is it not true? Can I give an example? Why does it work? How might I convince my group members that my conjecture is correct?
- And you will frequently be asked to explain your reasoning – how you were thinking while you were solving a problem, why you think students will solve problems in a particular way, and why you think some methods for solving problems work better than others. Developing good explanations that are convincing to others is the best way to know that you understand.
- My role extends beyond direct instruction to support and facilitate your engagement with and exploration of the material.

Course Requirements and Grading: (Note: all course requirements meet the program-level goal 4, understands the central concepts, tools of inquiry, and structures of the discipline(s) he or she teaches, particularly as they relate to the NJ Student Learning Standards (NJ PLS #4: Content Knowledge))

3 exams	45%
Reflections/exit tickets and class participation	10%
homework (recorded presentations posted on Canvas)	20%
final exam	25%

Grading scale

A	94 – 100%
A-	90 to < 94%
B+	87 to < 90%
B	83 to < 87%
B-	80 to < 83%
C+	77 to < 80%
C	73 to < 77%
C-	70 to < 73%
D+	67 to < 70%
D	60 to < 67%
F	< 60%

Class policies:

1. Additional assignments for extra credit will not be given.
2. The instructor must be contacted prior to any due date to negotiate alternative arrangements if necessary. For each day an assignment is late, it will drop 10% of the grade. If you need an extension, please contact me prior to the due date to discuss.
3. Students are responsible for any work that is missed. Canvas should be checked for any updates.
4. Make-up exams will be given for excused absences in which case I must be notified within 24 hours of the missed exam. Documentation may be required. Make-up exams will generally be harder.

Homework problems (Canvas Discussion):

Homework will be assigned frequently. It will not be collected and graded. Instead, we will use Canvas to record videos of homework solutions. You will each choose 4 or 5 homework problems over the course of the semester to solve and present on Canvas. (You will not be permitted to sign up for more than one problem until everyone has an opportunity to do a problem in that round.) A rubric for the homework solution presentations will be distributed. Keep in mind that your classmates will be watching your videos to try to understand how to solve the homework problems that they weren't able to solve so you should explain the solution clearly. Canvas provides the ability to respond to videos so you will be able to ask each other questions about the solutions. If there is a solution with many questions, it may be discussed in class. In addition to working together in class, you are encouraged to work together outside of class on problems. However, the final expression, the answer to a question, the solution and explanation to a problem, are matters for individual action to show that each person comprehends the matter at hand in their own way. Work that is merely copied will not be tolerated.

Exams:

See course outline for dates. The final exam will be given during final exam week at the scheduled time and will be cumulative.

Learning activities:

Learning activities will consist of a combination of lectures, explorations, group work, participation in class discussions, readings, reflections, and recorded presentations.

Group work:

A major part of this course will be spent working in small groups. We will form these groups near the beginning of the semester with the option at some point during the semester to switch them around. Naturally, if anyone has a major problem working in a particular group, you may tell me and we will switch the groups. In addition to working together in class, you are encouraged to work together outside of class on problems. However, the final expression, the answer to a question, the solution and explanation to a problem, are matters for individual action to show that each person comprehends the matter at hand in their own way. Work that is merely copied will not be tolerated.

Class participation:

Participation in the course takes many forms, including working in small groups, participating in whole class discussions, asking questions, listening, presenting your work, preparing for class by completing homework and assigned readings, etc. To be successful in the course, it is important for you to be an active participant. Written reflections/exit tickets done on Canvas will also factor into the class participation grade. Below is a rubric for class participation:

	Unsatisfactory	Basic	Proficient	Distinguished
	0-4	5-6	7-8	9-10

<p style="text-align: center;">Class Contributions and Preparation</p> <p><i>*Participation is reduced by absence or tardiness</i></p> <p>**Contributing positively to a class also means listening to, acknowledging, and respecting opinions of others.</p>	<p>Student almost never contributes to class by offering ideas and asking questions, and/or is almost never prepared for class.</p>	<p>Student sometimes contributes to class by offering ideas and asking questions, rarely volunteers to present work, and/or is inconsistently prepared for class.</p>	<p>Acts professionally, often proactively contributes to class by offering ideas and asking questions, volunteers to present work, and is usually prepared for class.</p>	<p>Acts professionally at all times, consistently and proactively contributes to class by offering ideas and asking questions sometimes more than once per class, consistently volunteers to present work, and is always prepared for class.</p>
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Note that your participation grade will not be affected if you are ill and cannot attend class. Please do not come to class if you are sick! A zoom option will be available for anyone who is sick and/or must isolate. Please note that zoom is ONLY to be used in cases of illness or isolation. Otherwise, you are expected to attend class as usual.

- **Important note:** *Incorrect solutions* can add value to a problem. Your participation grade is therefore not based on the correctness of your work.

4th Hour

In this class, the deep learning outcomes associated with TCNJ's 4th hour are accomplished by a series of rigorous educational assignments that extend beyond the typical scheduled class time. These include additional out-of-class reading, writing, and homework problems.

Tentative Course Schedule (subject to revision as needed)

Dates	Content	Reading
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9/1	Foundations for Learning Mathematics	Section 1.1
9/6 – 9/8	Foundations for Learning Mathematics Sets, Number Systems	Sections 1.1 & 1.2
9/12 – 9/15	Whole numbers and place value Addition of whole numbers	Sections 1.2 & 3.1
9/19 – 9/22	Subtraction of whole numbers	Section 3.2
9/26 – 9/29	Multiplication of whole numbers Exam 1	Section 4.1
10/3 – 10/6	Multiplication and Division of whole numbers	Sections 4.1 & 4.2
10/10 – 10/13	Fall break Division of whole numbers	Section 4.2
10/17 – 10/20	Fractions	Section 2.2
10/24 – 10/27	Fractions	Section 2.2
10/31 – 11/3	Addition and subtraction of fractions Exam 2	Sections 3.3 & 3.4
11/7 – 11/10	Addition and subtraction of fractions Multiplication and division of fractions	Sections 3.3, 3.4, & 4.3

11/14– 11/17	Multiplication and division of fractions	Section 4.3
11/21 – 11/24	Decimals and real numbers Thanksgiving Break	Section 2.3
11/28 – 12/1	Decimals and real numbers	Section 2.3
12/5 – 12/8	Algebraic Thinking (if time permits) Exam 3	Parts of Chapter 6
12/12 – 13	Reading Days Review for final exam	
12/14 – 20	Final exam scheduled by Records & Registration	

SELECTED TCNJ POLICIES

Please note that the “[TCNJ Student Support Resources and Classroom Policies](#)” webpage contains all of the relevant policies mentioned below and others that govern the classroom.

Final Assessment

All courses are to have a final assessment and no final assessments will be scheduled before the final exam period. Students are expected to take their final assessments in the time blocks assigned by Records and Registration. Students must notify the instructor two weeks before the final assessment if they have a conflict as defined by the Final Assessment and Reading Day policy.

Attendance

Students are expected to check the college calendar, and plan their course schedules and vacations so as to enroll only in those classes that they can expect to attend on a regular basis. Students are expected to participate in each of their courses through regular attendance at lecture and laboratory sessions, complete assignments as scheduled, and to avoid outside

conflicts, including for the final exam. It is further expected that every student will be present, on time, and prepared to participate when scheduled class sessions begin. In all circumstances, it remains the student's responsibility to initiate discussion about absence and arrangements for making up any missed work with each instructor.

Academic Integrity Policy

The College of New Jersey is a community of scholars and learners who respect and believe in academic integrity. This integrity is violated when someone engages in academic dishonesty. Complaints of student academic misconduct will be addressed and adjudicated according to the *Academic Integrity Procedural Standards*.

COMMITMENT TO DIVERSITY, EQUITY, INCLUSION, ACCESS AND BELONGING

The TCNJ community is composed of people with diverse backgrounds, perspectives, and experiences, and the college is committed to diversity, equity, inclusion, access and belonging. The college's Campus Diversity Statement can be viewed here: <https://diversity.tcnj.edu/campus-diversity-statement/>.”

CLASSROOM ENVIRONMENT AND COMMITMENT TO STUDENT SUCCESS, SAFETY AND WELL-BEING

The TCNJ community is dedicated to the success, safety and well-being of each student. TCNJ strictly follows key policies that govern all TCNJ community members' rights and responsibilities in and out of the classroom. In addition, TCNJ has established several student support offices that can provide the support and resources to help students achieve their personal and professional goals and to promote health and well-being. You can find more information about these policies and resources at the “TCNJ Student Support Resources and Classroom Policies” webpage here: <https://academicaffairs.tcnj.edu/tcnj-syllabus-resources/>.

Students who anticipate and/or experience barriers in this course are encouraged to contact the instructor as early in the semester as possible. The Accessibility Resource Center (ARC) is available to facilitate the removal of barriers and to ensure reasonable accommodations. For more information about ARC, please visit: <https://arc.tcnj.edu/>.”

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